holistic model, machine tool, power losses, error

# Jerzy JEDRZEJEWSKI<sup>1</sup> Wojciech KWASNY<sup>1</sup>

# KNOWLEDGE BASE AND ASSUMPTIONS FOR HOLISTIC MODELLING AIMED AT REDUCING AXIAL ERRORS OF COMPLEX MACHINE TOOLS

The paper presents the idea and need for the holistic modelling of machine tools. The complexity of such modelling is illustrated for a 5-axis machining centre. The need to model errors in the controllable axes of machine tools with rotational and linear motion drives is indicated. The physical dependences having a bearing on the state of deformations produced by temperature and dynamic forces, as well as the assumptions and computational models of shift generation in the high-speed bearing sets of spindles, and models of the behaviour of the ball screw are presented. Moreover, the results of experimental studies of the thermal behaviour of linear motor sets and guideways with rolling blocks are reported.

## 1. INTRODUCTION

The minimization of machine tool errors is of key importance for increasing the productivity of machine tools. This applies particularly to conventional precision machining, micromachining and nanomachining. The more demanding are the machining precision requirements, the more closely the error sources and their behaviour in the natural static, thermal and dynamic conditions need to be examined. Dealing separately with individual errors is insufficient in most cases. All the errors and their interactions should be analyzed jointly.

Modelling is employed to improve and optimize machine tools. The starting point for modelling is always the up-to-date theoretical and practical knowledge of the considered family of machine tools, covering their history and development. Computational analyses of machine tools focus mainly on accuracy disturbances and errors and their minimization and compensation.

In most practical cases, when measuring errors in dynamic states (characteristic of HSC machining), the total of thermal errors and errors due to dynamic loads is identified. For example, in the case of high-speed spindles, the sum of the axial thermal elongation and the spindle shift caused by centrifugal forces is determined [1],[2],[3],[4]. The same applies

<sup>&</sup>lt;sup>1</sup> Institute of Production Engineering and Automation, Wroclaw University of Technology

to positioning errors in the ball screw feed drives, in which case the structure deformation error caused by the forces of inertia of the masses being moved is added to the thermal error [5],[6],[7]. In order to holistically identify such errors, one needs to model them accurately on the basis of very deep knowledge enabling, one to take into detailed account all the physical phenomena involved. Such knowledge is acquired from both fundamental research and industrial research, conducted by the manufacturers and the investigators of the particular components of controllable axes.

The reduction of machine tool errors to errors in machine tool controllable axes greatly facilitates their minimization and compensation. Being aware of this, machine tool manufacturers put much emphasis on creating designs maximally geometrically and thermally axially symmetric. Thermal symmetry is achieved not only through the proper distribution of heat sources and masses and the shaping of heat exchange, but also through the selection and arrangement of points linking the machine tool housings. For this purpose, detailed holistic models are needed. The assumptions for and the acquisition of the necessary knowledge are analyzed in the subsequent parts of this paper. Recently research into the modelling of machine tool thermal errors has been undertaken by many research centres, particularly with regard to effective error compensation [8],[9],[10],[11]. Also machine tool manufacturers have been intensively studying this problem, aiming at increasing more precise error identification and compensation. For example, the number of compensated components of the spindle head rotation axis position error has been increased from 4 to 11, whereby the error has been reduced from 12µm to 4µm (courtesy of Y. Hanaki the President of OKUMA Co. - an unpublished paper delivered at the CIRP-PMI 2012 Conference).

## 2. HOLISTIC MODELLING ASSUMPTIONS

The starting point for computational analyses of machine tools is their post-assembly condition which meets the quality requirements with regard to the operating parameters. The requirements are specified by the manufacturer internal standards or the relevant ISO standards. The basic computational model must enable the numerical simulation and analysis of machine tool behaviour in the whole range of variation in the operating parameters. In many cases, simplifications are made and the computations are reduced to determining the machine tool error in a single point of the workspace and for a single spindle/tool position relative to the table/workpiece. However, for machine tool accuracy evaluation the determination of the deviations from the assumed positions/errors in threedimensional space is genuinely useful. A model enabling such error identification is highly complex. It must very accurately map the deformations caused by external and internal forces, the thermal deformations and their interactions and the dynamic loads and their combined effect on power losses and thermal strains. Only such a holistic model enables one to accurately identify the geometries/errors in transient states (Fig. 1). If the model also covers vibrations then full machine tool virtualization is possible. Therefore intensive research has been undertaken to develop fully integrated holistic models. A major constraint is the extreme complexity of the computational tasks involved and the still inadequate computing power of the commonly available computers.

It should be noted that in holistic modelling complex thermal phenomena are considered jointly with the impact of naturally variable dynamic loads on them. Because of the interdependence of the processes the loads also depend on the thermal state. This applies to both the high-speed spindle assembly with angular ball bearings and to rollingscrew gears. The neglect of the thermal and force interdependences in HSC machine tools is a gross simplification. However, many researchers make such simplifications when modelling, e.g., self-excited (chatter) vibrations.



Fig. 1. Main components of machine tool holistic modelling and optimization

Because of the introduced simplifications it is essential to finely tune (on the basis of temperature and thermal displacement measurements) the models used for modelling the thermal behaviour of the particular controllable axes of machine tools.

Finally, a separate problem is the complexity of holistic modelling. Only the main machine tool operating states or the entirety of the processes which occur in long time intervals of machine tool use can be modelled. In the latter case, modelling also covers precision disturbance processes, the arising of errors and their behaviour, error self-service and compensation, the arising and servicing of critical states and the behaviour of errors in the accepted intervals of their variation in any machine tool operating conditions (Table 1).

The models of errors arising in the above states and their variation must be experimentally verified. This provides the basis for the optimization of both accuracy and the whole operation process [12].

Real processes tak	king place in machine tool	Virtualization of machine tool behaviour
Processes of degradation and servicing	Identification of properties	Processes of modelling and optimization
Post-assembly service properties	Accurate identification of state of properties	Accurate models of properties
Disturbances	Identification of state sources and disturbance variation	Models of disturbance generation and behaviour in operating conditions
Geometry and motion errors in controlled axis	Identification of errors and their variation	Models of errors and their behaviour in operating conditions
Self-servicing	Identification of servicing process and properties	Self-servicing modelling
Error compensation	Identification of compensation process	Predictive models of error and compensation
Generation of critical states	Prediction of critical states	Predictive models of critical states
Servicing of critical states	Identification of repair processes	Models of repair processes and critical states
Permissible level of disturbances and errors	Continuous identification of state	Models of continuous error state monitoring

Table 1.	Identification	and virtualiz	zation of	processes t	aking p	lace in a	machine t	ools
					01			

In holistic modelling it is also essential to take into account intelligent processes, such as self-healing, in the form of, e.g., active cooling, influencing the rate of heat exchange and controlling the loading of the spindle and ball-screw rolling elements in the feed drive. If such intelligent processes are present, it is necessary to take them into account in order to accurately predict controllable axis errors which need to be compensated.

Figure 2 illustrates the modelling of the behaviour of a 5-axis machining centre in its controllable axes. Many of the modelling assumptions and test results presented in this paper are for this centre and a lathe centre.



Fig. 2. Holistic model of 5-axis machining centre

## 3. ROTATIONAL AXIAL DRIVE MODEL

For holistic modelling, it is vital to develop accurate models of spindle assemblies and ball screw assemblies. The assemblies have a direct bearing on machine tool precision and errors. The latter must be accurately predicted and compensated. Considering that today these are high-speed assemblies, motion dynamics must be taken into account in the modelling of the heat generation and heat transfer in them. The motion dynamics are characterized by great accelerations, sharp jerks and great centrifugal forces and gyroscopic moments acting on the rotating elements. This significantly affects power losses and the deformations of the spinning elements and their displacements. In the case of the loadbearing structures of housings, to a different degree insulated from the surroundings by closed spaces and shields, their dynamics are significantly affected by their thermal capacity or inertia. Attempts are made to reduce heat accumulation to the minimum. Nevertheless, it must be taken into account in precise modelling. Also all kinds of screens insulating the load-bearing structure of machine tool housings from external heat sources and from the heat generated by the machining process should be taken into account. The variation in heat generation in the course of machining is high and depends on the particular machining processes.

Spindle assemblies are modelled as heat sources with variable thermal output determined by changing hydrodynamic friction moment M<sub>0</sub>, the loads originating from the external and internal forces (in the form of moment  $M_1$ ) and the variable heat transmission conditions, especially under forced cooling. The basic relations through which  $M_0$  and  $M_1$ can be incorporated into the model are shown in Table 2 (see Annex). The numerical model takes into account the interdependence between the internal loads in the bearings and the thermal deformations of the spindle assembly components. It describes in minute detail the state of loading in transient states and so the variation in the thermal output of the bearings. In the same way the power losses in the bearings of the rotary and tilting tables in axes A and C are modelled (Table 3 - see Annex). The bearings (especially cross bearings) used in such tables (Fig. 3), with one row of rollers carrying radial loads and two rows carrying axial loads, are characterized by low sensitivity of power losses to changes in axial loading during heating. An increase in the axial force in one row of rollers is compensated by a reduction in load in the other row or in the perpendicularly set rollers whereby the total power losses are approximately constant. Thanks to this the computational procedure can be simplified.

The relations for power losses in torque motors and typical cooling conditions are shown in Table 4 (see Annex). The torque motors integrated with the axes of rotation, replacing the traditional designs of high-speed machine tool spindles, constitute variable large sources of heat. Owing to them increasingly higher rotational speeds – as high as 45 000rpm or even above 60 000rpm – can be reached. As the torque, the accelerations and the jerks are constantly increased, the motors ensure greater machine tool productivity and dynamic stability. The motors need to be intensively cooled in order to ensure the required operating characteristic, the proper operation of the spindle assembly bearings and the minimization of thermal errors in the axis of rotation.



Fig. 3. Types of tilting rotary table bearings

As already mentioned, in high-speed spindle assemblies with angular contact ball bearings, housed in a slidable sleeve tensioned with springs, shift significantly affects the position of the spindle tip. The causes of the shift are described in detail in [1]. The structure of the mathematical model for calculating the shift is shown in Fig. 4. A change in any of the model parameters disturbs the equilibrium of the forces acting on the rolling elements and races of the bearings. The equilibrium can be regained only by changing angles  $\alpha_0$  and  $\alpha_i$ . In reality, the balls seek a position in which the equilibrium takes place. In the model, this position and the appropriate angles are determined iteratively in two stages. In the first stage, it is assumed that the contact strains on the inner and outer races are equal and the condition:  $\alpha_0 + \alpha_i = 2\alpha_p$  is satisfied (Fig. 4), whereas in the second case, the condition is not satisfied. Sufficient accuracy of angle determined internal axial unbalance  $F_x$  which results in greater deflection of the springs and in a shift of the sleeve and consequently, in the displacement of the spindle tip as the rotational speed changes.

Knowing reactions  $R_i$ ,  $R_o$ ,  $R_a$  one can determine the effects which will ensue when any of the reactions changes as a result of external causes, such as:

- a change in the interference between the spindle and the inner ring,

- heat-induced changes in the diameter of the rings and the balls,

- spindle diameter change, and so on.

Experimental studies carried out by the authors show that at 45 000rpm the shift for a spindle with a slidable sleeve exceeds the thermal elongation. The results obtained using the above model have been corroborated many times by tests carried out on experimental machine tool prototypes.



Fig. 4 Comprehensive mathematical model of loads in high-speed bearing: a) algorithm of first stage, b) equilibrium of forces, c) methodology of searching for equilibrium angles

### 4. LINEAR FEED AXES

#### 4.1. FEED DRIVE WITH BALL SCREW

Today linear feed axes are expected to ensure possibly highest and uniform speeds, high dynamics (even above 2g; the average being in a range of 0,7-0,9) and precisely controllable high accuracy of positioning headstocks, tables and carriages. The latter parameter of the linear drive characterizes the structure very well since it comprises all the thermal interactions, the force interactions and the interactions between them. Thus it takes into account the thermal elongation of the drive components as they heat up, their deflections caused by the pulling force and the friction forces and the dependences between

power losses, the stiffness of the bearing carriages, the stiffness of the connection between the ball nut and the screw, and the stiffness of the ball screw itself.

Therefore when considering drives with a ball screw one should take into account several aspects which determine the intensity and variability of the heat sources (the motor, the bearings, the nut/screw, the guideways), the thermal displacements and the displacements caused by pulling force  $F_{pull}$ , force R tensioning the ball screw and the friction forces on the guideways (Fig. 5). These are:

- the nut preload,
- the nut/screw friction,
- the guideway friction,
- the bearings friction,
- the screw pretension,
- the inertia,
- the gravity,
- the rigidity,
- the axis angle,
- the cooling,
- the outer force.
- the environment.



Fig. 5. Ball screw feed axes: a) integrated feed assembly, b) drive components

Besides speed, the following parameters can change as a result of drive operation:

- the moment and power of friction in the nut, changing with the pulling force determined by the variable friction forces and the forces of inertia of the shifted masses as the carriage moves to the left and right or up and down (Table 5, Fig. 6). Table 5 (see Annex) does not include additional causes of the variation in the friction moment, stemming from changes in the rotational speed of the ball screw. The immediate cause can be the rolling friction coefficient and changes in the contact angles of the balls in the nut assembly [18],



Fig. 6. Power loss variation in Ø36 nut during acceleration, travelling at constant speed of 0,5m/s and braking

- the axial force loading the bearings, changing with the thermal elongation of the ball screw,
- the power losses in the bearings, changing with the axial force in the ball screw (Fig. 7),
- the ball screw stiffness changing with the position of the nut, directly affecting the positioning error (Fig. 8),
- the stiffness of the bearings depends on the variable axial force in the ball bearing and on the position of the nut, having a bearing on the axial displacements of the nut,
- the force and power of friction on the guideways, changing with speed and determining the coefficient of friction and the distribution of pressures on the guideways (Fig. 9).



Fig. 7. Effect of actual ball screw tension on bearing power losses



Fig. 8. Axial stiffness of preloaded ball screw for fixed-free and fixed-fixed bearing systems



Fig. 9. Moving heat sources impacting bed guideways during motion of carriage: a) load distribution depending on motion phase, b) guideway areas which heat up, c) exemplary changes in friction power for upper guideway Q1 and lower guideway Q4

Similarly as for the slide guideways, the thermal output of the rolling blocks was modelled (Table 6 see Annex). The distribution of the forces loading the particular blocks stems only from the distribution of the masses and from the forces of inertia acting during acceleration and braking. The efficiency of the rolling systems of linear motion is very high (>99%), mainly owing to the very low coefficient of rolling friction (ranging from 0,002 to 0,003 for rolling elements and from 0,005 to 0,01 for roller elements). Figure 10 shows the rise in temperature measured on the rolling block housing and on the guideway during the operation of a medium-sized milling centre. Actually, this is the result of the combined action of the heat generated in the rolling blocks and the heat coming from the linear motor (the heat impacts could not be separated).



Fig. 10. Rise in temperature of guideway in medium-sized 5-axis machining centre for: a) maximum feed rate, b) 40% of maximum feed rate

In general, the problem of heat load modelling comes down to the modelling of heat sources with variable output, combined with the instantaneous temperatures and thermal elongations of the ball screw and with the interactions connected with the pulling forces generated by the motor to overcome the variable motion resistances, the inertial forces and the gravity forces.

It should be mentioned that today the maximum feed rates achieved using the ball screw and the direct linear motor are as high as 50-60 m/min and 80 m/min, respectively.

#### 4.2. FEED DRIVE WITH LINEAR MOTOR

Direct feed drives (Fig. 11c) are a.c. motors with windings in the primary part and permanent magnets (in the secondary part), fixed to a stationary element, such as the machine tool bed, the carriage body, etc.



Fig. 11. Temperature rise for linear motor working in medium-sized 5-axis machining centre at: a) maximum feed rate, b) 40% feed rate, c) linear drive components

The motor design, e.g. the size of the air-gap between the primary part and the secondary part, the magnet materials, the masses moved, the supply voltage and the forces loading the motor during feeding (including the cutting forces), determines power losses.

The motors must be very intensively cooled, especially the part with windings, in which nearly 98% of the total heat is generated. The heat flux which, despite the working coolers, may penetrate into the machine tool load-bearing structure, is mainly determined by the efficiency and stability of the cooling systems. According to the manufacturers of the motors, the structure may heat up by a few degrees as a result. This has been confirmed by measurements of the rise in temperature of the primary and secondary parts in a medium-sized milling centre (Fig. 11a,b). The temperature of the stationary part, impacting here the housing, increases maximally by 2°C and that of the moving part being cooled, by as much as 5°C.

Such practical knowledge can be directly exploited to model the thermal impacts of linear motors on the adjoining machine tool components.

### 5. DISCUSSION OF HOLISTIC MODELS IMPLEMENTATION

The presented models of heat generation in the rotational and linear drives of machine tools interrelate the thermal output of the sources with the temperatures, forces and geometric dimensions, which continuously change in the course of operation.

Through the thermal dimensional changes in the diameters of the races and the balls and through the axial thermal dimensional changes of the spindle unit components the changes in temperature affect the internal load in the bearings.

Similarly in the feed drives, the changes in temperature, which occur in thermally nonstationary states, cause changes in ball screw preload, which in turn lead to changes in the force loading the ball screw bearings and the supports of the bearings, with all the thermal and stiffness consequences. As a result, the power losses in the bearings and in the motor driving the ball screw as wells as the heat and force induced deformations (including the positioning error) change.

When the models are implemented in FEM computational systems, the modelled object is loaded with stationary or moving heat sources with their output changing depending on the instantaneous state of stress, the temperatures and the heat and force induced deformations in the computed structure. As a result of heat distribution and the force and heat induced deformations of the whole structure (computed by the FEM software), interactions between the machine tool units take place.

As mentioned earlier, the objective towards which one should move is a holistic model of the machine tool. Because of the two constraints:

- the insufficiently comprehensive models of the phenomena and interrelations involved in the operation of the main machine tool units (insufficient knowledge base),
- the inadequate computing speed of the available FEM programs, especially in the case of a larger number of simultaneously operating complex units,

the objective has to be reached in stages.

The Wroclaw University of Technology research team, to which the present authors belong, has been working on accurate models and their integration for many years. To date, three holistic model concepts for:

- a machine tool with one operating, structurally complex headstock,
- a machine tool with one operating electrospindle with different bearing configurations,
- a machine tool with one operating feed drive with a ball screw have been developed.

Some of the results relating to displacements and temperature distributions, obtained using the first two holistic models have already been published [15],[22],[23],[24]. Further research on holistic models covering the simultaneous operation of many linear and rotational drives is underway. This means that there is a knowledge base sufficient to integrate the correlations describing power losses, heat transfer, including forced cooling in motion (moving heat sources) conditions [not published yet], deformations and displacements (induced by heat as well as by external and centrifugal forces), accelerations and jerk in the particular modules of the controllable axes.

The models developed and validated for specific 5-axis machining centres and turning centres are helpful in the computer simulation of errors and in the creation of their simplified models implemented in compensation procedures, not only in the central point, but also in space [not published yet].

Currently the research is focused on refining the correlations to be implemented in the models and integrating them into the FEM-ABAQUS system in order to ensure natural interactions.

#### 6. CONCLUSION

The models presented in this paper can be successfully used in the holistic modelling and computational simulation of high-speed machine tool axial drive modules for improving their behaviour and error compensation. They are being constantly upgraded and fine-tuned as new materials are introduced and rotational speeds and feed rates increase.

The precision of holistic modelling depends on the accuracy of the relations describing the phenomena involved and their integration into the FEM system used. Today the holistic modelling of machine tools has become a key research problem.

#### REFERENCES

- [1] JEDRZEJEWSKI J., KWASNY W., 2010, Modelling of angular contact ball bearings and axial displacements for high-speed spindles, CIRP Annals Manufacturing Technology, 59/1, 377–382.
- [2] MAYR J., JEDRZEJEWSKI J., UHLMANN E., DONMEZ M.A., KNAPP W., HARTIG F., WENDT K., MORIWAKI T., SHORE P., SCHMITT R., BRECHER C., WURZ T., WEGENER K., 2012, *Thermal issues in machine tools*, CIRP Annals – Manufacturing Technology, 61/2, 771–791.
- [3] JEDRZEJEWSKI J., KWASNY W., 2012, *Holistic precision error model for 5 axis HSC machining centre with rotating rolling units in direct drives*, Procedia CIRP, 4, 125-130.
- [4] JEDRZEJEWSKI J., KWASNY W., 2012, Integrated modelling of axial motion error sources for 5-axis precision machining centre with direct drives, MM Science Journal, November, 346-351.
- [5] WU C.-H., KUNG Y.-T., 2003, *Thermal analysis for the feed drive system of a CNC machine center*, International Journal of Machine Tools & Manufacture, 43, 1521–1528.
- [6] HEISEL U., KOSCSAK G., STEHLE T., 2006, *Thermography-based investigation into thermally induced positioning errors of feed drives by example of a ball screw*, CIRP Annals Manufacturing Technology, 55/1, 423–426.

- [7] KIM J.-J., JEONG Y.-H., CHO D.-W., 2004, *Thermal behaviour of a machine tool equipped with linear motors*, International Journal of Machine Tools & Manufacture, 44/7-8, 749–758.
- [8] YANG H., NI J., 2005, Adaptive model estimation of machine-tool thermal errors based on recursive dynamic modelling strategy, Machine Tools & Manufacture, 45, 1-11.
- [9] TUREK P., JEDRZEJEWSKI J., MODRZYCKI W., 2010, *Methods of machine tool error compensation*, Journal of Machine Engineering, 10/4, 5-26.
- [10] SCHWENKE H., KNAPP W., HAITIEMA H., WECKENMANN A., SHMITT R., DELBRESSINE F., 2008, Geometric error measurement and compensation of machines – An update. CIRP – Manufacturing Technology, 57, 660-675.
- [11] CREIGHTON E., HONEGGER A., TULSIAN A., MUKHOPADHYAY D., 2010, Analysis of thermal errors in a high-speed micro-milling spindle, International Journal of Machine Tools & Manufacture, 50, 386–393.
- [12] MEKID S. (editor), JEDRZEJEWSKI J. at al., 2009, Introduction to precision machine design and error assessment, CRC Press, Taylor&Francis Group, ISBN 978-0-8493-7886-7.
- [13] PALMGREN A., 1964, Grundlagen der Walzlagertechnik, Francklische Verlagshandlung, Stuttgart.
- [14] JEDRZEJEWSKI J., KWASNY W., STRAUCHOLD S., 1991, Numerical modelling of the thermal behaviour of spindle bearing assemblies for precise machine tools, Proc. of CIRP Conf. on PE&MS, 525-536, ISBN 75618-0280-3/TH14.
- [15] BLAZEJEWSKI A., KWASNY W., JEDRZEJEWSKI J., GIM T.-W., 2010, Modelling thermal deformation of tilting rotary table with direct drive system, Journal of Machine Engineering, 10/4, 18-32.
- [16] LUNDBERG G., 1949, Cylinder compressed between two plane bodies, SKF Reg., 4134.
- [17] ETEL, Torque Motors Handbook, 2008, Etel Motion Technology, 902/Ver. D.
- [18] VERL A., FREY S., 2010, Correlation between feed velocity and preloading in ball screw drives, CIRP Annals Manufacturing Technology, 59/1, 429-432.
- [19] NSK, Precision machine components, 2002, Catalogue, No.E9008a 2011 Z-9, NSK Motion & Control.
- [20] HIWIN, Motion control and system technology, 2010, Catalogue, FORM S99TE16-1003, HIWIN Motion Control & Systems, Taiwan.
- [21] THK, Caged Ball LM Guide, 2007, CATALOG No.356-2E, THK Co., Ltd. Tokyo, Japan.
- [22] JEDRZEJEWSKI J., KOWAL Z., KWASNY W., MODRZYCKI W., 2004, Hybrid model of high speed machining centre headstock, CIRP Annals Manufacturing Technology, 53/1, 285–288.
- [23] WINIARSKI Z., KOWAL Z., KWASNY W., HA J-Y., *Thermal model of the spindle drive structure*, Journal of Machine Engineering, 10/4, 41–52.
- [24] JEDRZEJEWSKI J., KOWAL Z., KWASNY W., MODRZYCKI W., 2005, High-speed precise machine tools spindle units improving, Journal of Materials Processing Technology, 162–163, 615–621.

# ANNEX

Calculated quantity	Basic formulas	Source
Total friction torque Torque due to applied load	$M = M_o + M_1$ $M_1 = f_1 P_1 d_m, \text{Nmm}$ $f_1 - \text{factor related to bearing type and load}$ $P_1 - \text{equivalent load dependent on external bearing load forces}$	Palmgren [13]
Torque due to applied load	$d_m - \text{ mean diameter of bearing (mm)}$ Bearing operation in negative clearance range $M_{m} - f \Gamma d_{m} \text{ Nmm}$	Jedrzejewski [14]
	$\Gamma \Gamma_{m} = \int_{1}^{1} \Gamma_{m}$ , Ninn $\Gamma_{m} = $ equivalent load dependent on external bearing load forces and on internal forces	
Equivalent load	$\Gamma = \begin{vmatrix} F_r & \text{if } F_r > P_1 \text{ and } > F_e \\ P_1 & \text{if } P_1 > F_1 \text{ and } > F_e \\ F_e & \text{if } F_e > F_1 \text{ and } > F_r \end{vmatrix}$ $F_r - \text{external radial force acting on bearing}$ For roller bearing $-F_r = 5 z I^{1.1} I^{0.9}$ N	Jedrzejewski [14]
	$L_r$ – absolute value of negative radial clearance, $\mu$ m $L_w$ – roller length, mm z – number of rolling elements For ball bearing – $F_e = 0.8zL_r^{1.5}d_k^{0.5}$ , N	
Torque due to lubricant viscous friction	$d_k$ – ball's diameter, mm <b>rich/jet lubrication</b> $M_0 = 10^{-7} f_o (\nu n)^{2/3} d_m^{-3}$ , Nmm $f_o$ – factor related to bearing type and lubrication method $\nu$ – kinematic viscosity of lubricant (mm <sup>2</sup> /s) n – rotating speed of bearing (rpm), <b>minimal lubrication</b>	Palmgren [13]
	$M_0 = 10^{-7} \Psi f_o (\nu n)^{1/3} d_m^{-3}$ , Nmm $\Psi$ – coefficient of minimal lubrication dependent on type of bearing	Jedrzejewski [14]

Table 2.	Friction	torque	in s	spindle	bearings

Table 3.	Friction	torque	of tiltin	g rotary	table	bearings
				8 ,		

Calculated quantity	Basic formulas	Source
Total friction torque	Conventional and cross axial/radial cylindrical bearings $M = \sum_{i=1}^{k} M_{0i} + \sum_{i=1}^{k} M_{1i},$	Palmgren [13]
	$M_{0i}, M_{1i}$ – friction torque, k – number of roller rows k=3 – conventional bearing; k=2 – cross bearing For $\delta_p < \Delta X_{max}$	
Torque due to applied load	$M_1 = M_1(1^{st} row) + M_1(2^{nd} row) = \text{constant}$ $\Delta X - \text{change in race distance due to heating up of structure}$ $\delta_p - \text{strain caused by bearing preload}$	Blazejewski [15]

	For conventional bearing	
Strain	$\delta_{\max} = 0,079 \cdot \frac{Q_o^{0.9}}{L_w^{0.8}}, \mu m$	Lundberg [16]
	For cross bearing	
Strain	$\delta_{\max} = 0,079\sqrt{2} \cdot \frac{Q_o^{0.9}}{L_w^{0.8}}, \mu\mathrm{m}$	Blazejewski [15]
	$Q_o$ – roller load:	
	For axial row	
	$Q = \frac{P_w}{N}$ , N	
Roller load	$z_o$ Z	
	For cross bearing	
Roller load	$Q_o=rac{\pi P_w}{2z}$ , N	Blazejewski [15]
	$P_w$ – axial bearing load, N	
	z – number of rollers in one row,	
	$L_w$ – roller length, mm	

Table. 4. Power losses in rotary direct drive motors

Calculated	Basic formulas
quantity	
	Total continuous power $P_c$ to be dissipated by coil
D	$P_{C_{(\Theta=130^{\circ}C)}}$ , W (in catalogue data sheets)
Power	15 $P_{a} I_{a}^{2}$
dissipation by	$Pc_{(\Theta \neq 130^{\circ}C)} = \frac{1.5 \cdot RC \cdot RC}{(100 \times 10^{\circ}C)}, W$ ETEL [17]
con	$\left(\frac{Tc}{T_{RMS}}\right)^2$
Torque	Tc - continuous torque for coil temperature of 130°C, Nm (in catalogue data sheets)
Current	<i>Ic</i> - continuous current for coil temperature of 130°C. Arms
	(in catalogue data sheets)
Resistance	Rc – resistance of windings at actual working temperature,
	$Rc = R_{20} (1 + (0,00392 \cdot (\Theta - 20))), \text{ Ohm}$
	$R_{20}$ – catalogue electrical resistance at 20°C, Ohm
	$\Theta$ – actual temperature of windings, <sup>o</sup> C
	$T_{RMS}$ - equivalent torque for whole cycle
Equivalent torque	$T_{RMS} = \sqrt{\frac{\sum_{i=1}^{N} T_i^2 \cdot t_i}{t_{cycle}}}$ , Nm
	$T_i$ - torque values for step, N - number of steps $t_i$ - step duration, $t_{cycle}$ = complete motion time
Power	
dissipation by	$P_{rotor} = 0.02 P c_{(\Theta \neq 130^{\circ} C)}$
rotor	Pc
Water flow	$Fw = 0.0143 \cdot \frac{\Gamma C_{(\Theta \neq 130^{\circ}C)}}{\Delta Tw}, 1/\min$
	$\Delta T_w$ - water temperature difference between input and output coolant
Pressure drop	$\Delta p = \Delta p_{(cata \log ue)} \cdot \frac{Fw}{Fw_{(cata \log ue)}}, \text{ bar}$

Calculated quantity	Basic formulas	Source
Basic torque of preloaded nut-ball screw connection	$M_p = \frac{K \cdot F_p \cdot L}{2000\pi}$ , Nm	NSK [19]
	$F_p$ – preload, N; $L$ – lead, mm	
	$F_p \leq \frac{F_a}{2.8}$	
	$F_a$ – equivalent external axial load	
	K – preload torque coefficient	
	$K = \frac{0.05}{\sqrt{\frac{L}{\pi d_m}}}$	
Friction torque during	$d_m$ – screw shaft pith circle diameter, mm	
acceleration	$M_a = J \cdot \varepsilon \frac{(1-\eta)}{\eta}, Nm$	HIWIN [20]
	$\eta$ – ball screw efficiency	
	$\boldsymbol{\varepsilon}$ – angular acceleration, rad/sec <sup>2</sup>	
	$\varepsilon = \frac{2\pi}{60t_a} \left( rpm_{stage2} - rpm_{stage1} \right)$	
	$t_a$ – acceleration rise time, sec	
Load inertia	J – load inertia	
	$J = W \cdot \left(\frac{L}{2000\pi}\right)^2, \text{ kg m}^2$	NSK [19]
Power losses	W- table and workpiece weight, kg	
assembly	$P = \frac{\left(M_p + M_a\right)n}{9.55}, \text{ W}$	
	<i>n</i> -rotational speed of ball screw. rpm	

Table 5. Power losses in nut-ball screw connection

Table 6. Power losses in rolling guideways

Calculated quantity	Basic formulas
	Ball and roller LM systems
Power losses	$P = (\mu \cdot F_m + f) \cdot v, \mathbf{W}$
	$\mu$ - friction coefficient, $\nu$ - speed of motion, m/s
	$F_m$ - average load, N, $f$ -, frictional resistance of seals, N
Average load of block	$F_m = \sqrt[3]{\frac{1}{L} \cdot \sum_{i=1}^n (Fw_i^3 \cdot L_i)}$
	$F_i$ – varying load, $L$ – total distance travelled,
Total load	$L_i$ – distance travelled under load $F_i$
of block	$Fw = F_o + k \cdot f_w \cdot F_{out},  \text{THK [21]}$
	$F_o$ – preload, $F_{out}$ – external load,
	k – external load factor
	$f_w$ – dynamic load factor