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REVIEW OF POTENTIAL ADVANTAGES AND PITFALLS OF NUMERICAL SIMULATION OF SELF-EXCITED VIBRATIONS

Machining stability is one of the most important factors influencing the geometrical and dimensional accuracy of the machined parts. Regenerative chatter is a major limitation to the productivity and quality of machining operations due to poor surface finish and faster tool wear. In general there are two methods of stability analysis: solution of differential equations of the system in frequency domain or numerical simulation in time domain. Fast and easy calculations in the frequency domain are possible using a simplified linear model of cutting process. Important limitations of these methods are difficult or impossible considering of changes of dynamic cutting force coefficients and dynamic characteristic of a process. Numerical simulation has not these limitations and regards many specific phenomena of the cutting processes, therefore it is often used in the stability analysis. The paper presents main advantages of numerical simulation, which differentiates it from the analytical solutions, as well as some inevitable difficulties and limitations.

1. INTRODUCTION

Self-excited vibrations occurring in the cutting processes are a major limitation for the achievable performance, machining quality, tool life and durability of machine tools. Hence there is a need of stability limit prediction allowing selection of chatter-free cutting parameters. Despite the real machine-tool-workpiece structure (MS) has a very complex structure with many degrees of freedom, for most applications, such as turning and milling, it can be reduced to a multimodal system with two degrees of freedom [8],[21],[27]. It still allows for consideration of two main causes of self-excited vibrations in machining, which are mode coupling and regenerative effect. Model of such system for milling is presented in Fig. 1, and general block diagram of a system is presented in Fig. 2. Differential equation of the open system can be expressed in a matrix form as:

$$MP'' + CP' + KP = F \tag{1}$$

where M, C, K – matrixes of masses, damping and stiffness of the machine-tool-workpiece structure (MS), F – vector of the cutting forces in the MS coordinate system, P, P', P'' – vectors of displacements, speeds and accelerations in the MS system.

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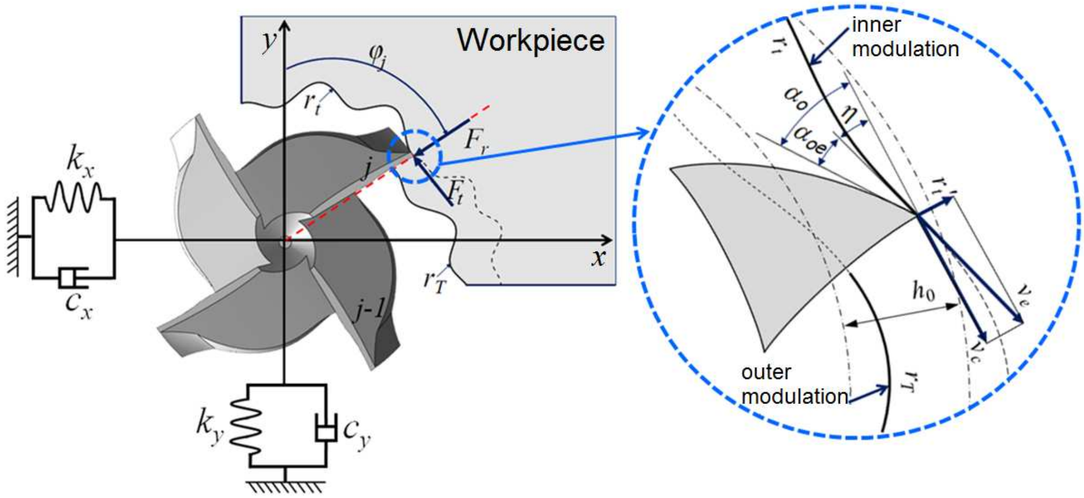

Fig. 1. Dynamic model of 2-DOF system in milling

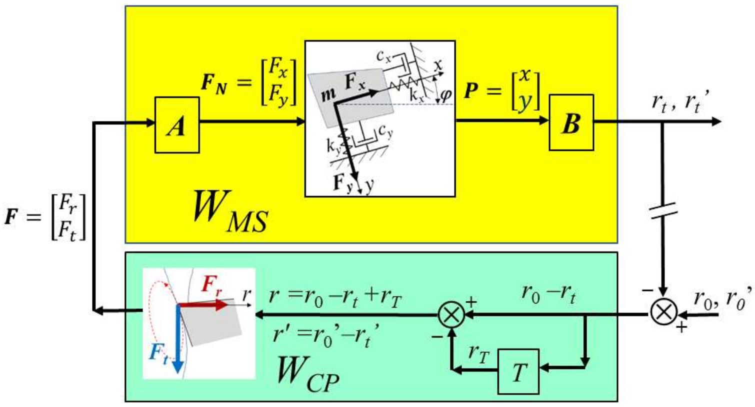

Fig. 2. Block diagram of dynamic 2-DOF system

Blocks shown in Fig. 2 are described by transfer functions defined as the relation between Laplace transforms of the inputs and outputs. Input signals of the cutting process (CP) are displacements and displacement speeds in the radial direction (r and r', see Fig. 1). The dependence of the cutting forces on vibrations in tangential direction t is usually ignored. Thus the variable cutting force components F_r and F_t in the CP coordinate system depend on dynamic changes of the uncut chip thickness caused by relative displacement between the workpiece and the tool in radial direction (inner modulation of the uncut chip thickness r_t) and the machined surface waviness left during the previous pass (outer modulation r_T) and the velocity of these displacements (r_t'). Vector of these cutting forces $\boldsymbol{F}$ is the output of the cutting process and the input of the machine-tool-workpiece structure MS (Fig. 2). Forces F_r and F_t (vector $\boldsymbol{F}$) are projected to the x, y directions in the MS coordinate system by the $\boldsymbol{A}$ matrix. Obtained forces F_x and F_y, (vector $\boldsymbol{F_N}$) cause vibration of the machine-tool-workpiece structure in x and y direction (vector $\boldsymbol{P}$). Those vibrations after projection to the r direction (by matrix $\boldsymbol{B}$) become the output signal of the MS.

Stability analysis is usually based on analytical or numerical solution of equation (1) in the frequency domain [16]. Despite the convenience of stability limit calculation the main disadvantage of these methods is the inability (or very high difficulty) to consider machine-tool system characteristic changes in a space and time, especially in case of complex non-linear characteristics of the cutting process. These limitations stimulate attempts of stability analysis based on time domain numerical simulation by many research centers.

In a single iteration of a typical algorithm of numerical simulation (Fig. 3) the following steps can be identified:

1) calculation of the current displacements (x_i, y_i) and velocities (x_i', y_i') for the each vibration mode i separately, and subsequently summed up,
2) projection of displacements and velocities of the system (x, y) to the r direction, individually for each segment of the cutting edge, and storing the displacements r as an outer modulation r_T used in the next tool pass,
3) determination of the variable force components F_r and F_t in the cutting process coordinates based on the assumed model dependences of these forces on r_t , r_t' and r_T,
4) projection of the F_r and F_t forces to the F_x and F_y forces and summing them up along the cutting edge.

The first step contains the main difference between stability analysis in frequency domain and numerical simulation in the time domain prediction of the vibration progress. In fact, it contains the basic algorithm of the simulation. Each step presented above and in Fig. 3 exhibits many possibilities, which differentiates numerical simulation from the analytical solutions, as well as some difficulties and limitations which are discussed in the following paper sections.

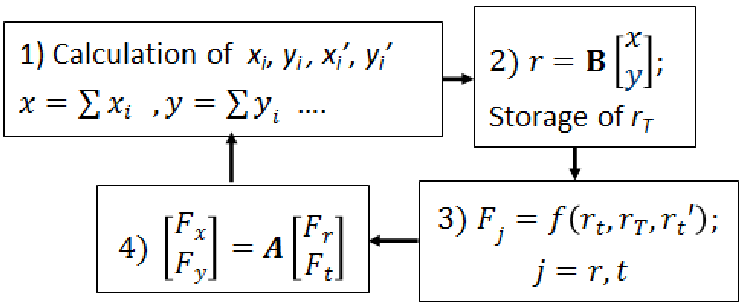

Fig. 3. A single iteration algorithm of numerical simulation of self-excited vibrations

2. DETERMINATION OF THE SYSTEM DISPLACEMENTS

There are two main approaches to the problem of numerical simulation in the time domain. In both of them each of the vibration modes of the multimodal system is analyzed separately, and then the displacements are summed up. The first approach, the most commonly used, is based on the Runge-Kutta method for the approximation of solutions of ordinary differential equations, e.g. [22],[34]. The equation of motion of every single mode "i" of the system is given by:

$$m_{pi}\, p_i" + c_{pi}\, p_i' + k_{pi}\, p_i = F_p \tag{2}$$

where $p = x$ or y, while m_{pi}, c_{pi}, k_{pi} are the modal mass, the modal damping constant and the modal spring constant of the "i" mode of the MS system. Dividing the left- and right-hand side of the equation by the modal mass m_{pi} yields:

$$p_i" + 2d_{pi}\omega_{0pi}\, p_{iB}' + \omega_{0pi}^2\, p_{iB} = F_p/m_{pi} \tag{3}$$

where ω_{0pi} is the natural frequency, d_{pi} is the damping ratio in the direction of the p axis. From equation (3) the tool-workpiece relative displacement p can be obtained using a first or fourth order Runge-Kutta method.

The second approach, also often used, is the algorithm presented in [37]. Here first the acceleration in the present iteration is determined for each vibration mode. Then the displacement is obtained by double integration and using velocity and displacement in the previous iteration:

$$\begin{cases} p_i" = \left(F_{pB} - c_{ip}\, p_{iB}' - k_{ip}\, p_{iB}\right)/m_{ip} \\ p_i' = p_{iB}' + p"\mathrm{d}t \\ p_i = p_{iB} + p'\mathrm{d}t \end{cases} \tag{4}$$

where $p = x$ or y, index B – indicates forces, displacement and velocity in the previous iteration.

In both approaches the displacements and velocities are summed up for all vibration modes of the multimodal system in both directions:

$$p = \sum p_i\,,\;\; p' = \sum p_i' \tag{5}$$

This step is absent when one mode system is considered. The summation is plain arithmetic addition.

Then the cutting forces F_p can be calculated using displacement and velocity in present iteration and assumed cutting force model (see below).

3. DETERMINATION OF DYNAMIC CUTTING FORCES COMPONENTS

3.1. DEPENDENCE OF THE UNCUT CHIP THICKNESS AND WORKING CLEARANCE ANGLE ON RELATIVE TOOL-WORKPIECE VIBRATIONS

The source of the cutting force variation is changes of the uncut chip thickness (h) and working clearance angle α_{oe} determined by the working clearance angle α_{oe} (see Fig. 1):

$$\begin{cases} h = h_0 - r_t + r_T = h_0 + h_d \\ h_d = -(r_t - r_T) \end{cases} \tag{6}$$

$$\begin{cases} \alpha_{oe} = \alpha_0 - \eta \\ \eta \approx \tan\eta = r_t{}'/v_c \end{cases} \tag{7}$$

where h – uncut chip thickness, h_d – dynamic component of uncut chip thickness, h_0 – nominal (steady state) value of uncut chip thickness, α_{oe} – working clearance angle, α_o – orthogonal clearance angle, η – effective cutting speed angle.

When analyzing the complex cutting edge, forces acting on the small individual segment are considered separately and summed up. Transition from machine-tool system *x-y* to the cutting process system *r-t* (step 2 in Fig. 3) is a simple transformation (rotation) of the coordinate systems. It should be noted however, that while *x* and *y* displacement are related to the whole machine-tool system, during milling with helical flute cutter, displacements are projected on *r* and *t* directions for each individual part of cutting edge separately, as the angular position φ_{ij} of the segment depends on the angle of cutter rotation and the angle of twist of considered segment. Fig. 4 shows the scheme of angle φ_{ij} determination. A generalized geometric model for any complex shape of the cutter can be found in that of Engin and Altintas [10], however the principles stay the same. The displacement of the cutting edge segment in perpendicular direction to the corresponding workpiece surface and the speed of this displacement can be described as (Fig. 4):

$$\begin{cases} r_{tij} = x_i \sin\varphi_{ij} + y_i \cos\varphi_{ij} \\ r'_{tij} = x'_i \sin\varphi_{ij} + y'_i \cos\varphi_{ij} \end{cases} \tag{8}$$

$$\begin{cases} \varphi_{ij} = \varphi_{ni} + \varphi_{zj} \\ \varphi_{zj} = \frac{2z_j}{D \cot\lambda_s} \end{cases} \tag{9}$$

where i –iteration (time) index, j – index of considered segment distance from the cutter face, φ_{ni} – rotation angle of considered tooth on the cutter face, φ_{ij} – rotation angle of considered segment, z_j – distance of the segment from cutter face, λ_s – inclination angle of the cutting edge, D – cutter diameter.

Instantaneous uncut chip thickness of the considered segment of the cutting edge can be described as follows:

$$h_{ij} = f_z \sin\varphi_{ij} - r_{ij} + r_{Tij} \tag{10}$$

where f_z – feed per tooth.

Obviously, equation (10) remains valid only for angles φ_{ij} between engagement angle φ_1 and exit angle φ_2 (Fig. 4) [3],[18]. However, for unstable cutting conditions, the uncut chip thickness must be determined considering variations in the tool tip position caused by the dynamic behavior of the cutter [22]. As illustrated in Fig. 5 backside cutting can occur at a tooth rotation angle φ_{ij} located outside (φ_1, φ_2) region.

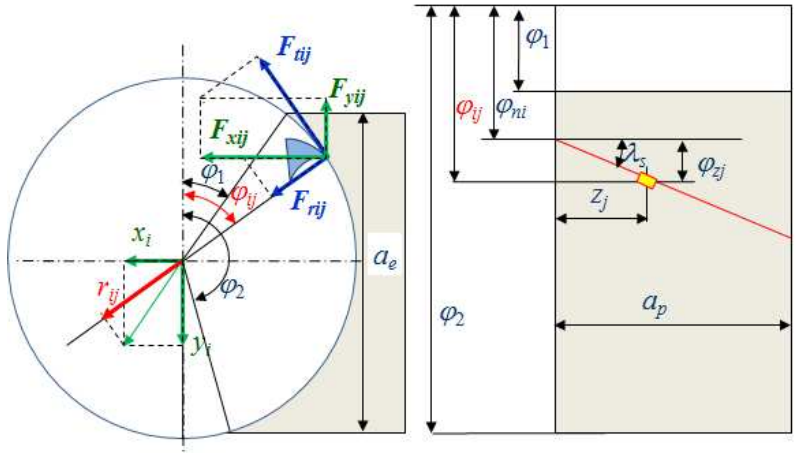

Fig. 4. Discretization of the cutting edge and the distribution of cutting forces for helical flute cutter

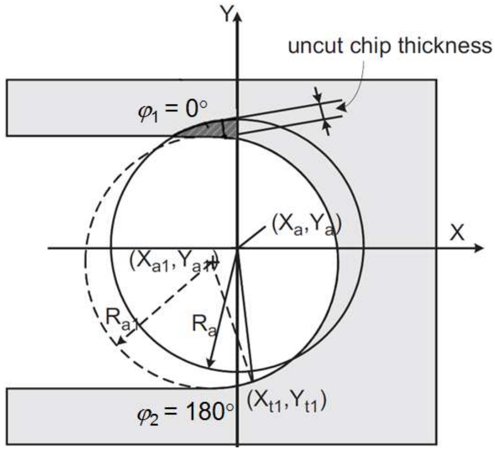

Fig. 5. Backside cutting effect for unstable cutting [22]

The cutting edge segmentation should depend on the tool shape and include the distribution of cutting forces on the cutting edge. Therefore, for milling tools with round corner (radial or ball end-mills) radial segmentation is preferred [40]. However division into equal segments is often used especially in the analytical method or numerical integration [13],[22]. In the case of end-mills the distribution along the axis of the elementary parts with the same shape and the same height is justified [25].

Numerical simulation also enables to consider the influence of the tool run out on the uncut chip thickness. It can be achieved by the assumption of circular (movement in one direction) or trochoidal tool trajectory [11]. In both cases, the geometric parameters of the process are constant along the tool path, but run out of the cutter is introduced to the each single point of cutting edge [9]. The most complex case specifies different trochoidal paths for each tooth, which means different active diameter for each tooth. Then the uncut chip thickness for the current tooth is the difference between the current and the previous tool path along the line segment connecting the center of the tool with the current cutting edge.

This approach is used instead of the low computational cost approximation, proposed by Martellotti [28]: $f_z\, sin(\varphi)$. Trochoidal tool path improves simulation effect of entry (φ_1=0^o) and exit (φ_2=180^o) tool zone, where uncut chip thickness is not equal 0 and it can achieve significant values when run out is considered.

3.2. STORAGE OF THE SURFACE CREATED IN PREVIOUS CUT

Considering the outer modulation of the uncut chip thickness requires storage in a computer memory the relative tool-workpiece displacements in radial direction r_t and introducing it to the calculation of instantaneous uncut chip thickness during the next cutting edge pass as r_T (Fig. 1, eq. 6). While it is quite simple for turning, it is much more complicated for milling. It can be realized by storage of all positions of the tool (and the workpiece if necessary) or storage of the generated surface (generating surface profile). Outer modulation can be reconstructed from the instantaneous position of the cutting edge segment, calculated for the *j*-th tooth, the *k*-th level along the tool axis and the *i*-th iteration step the tool having vibration and run out as [25]:

$$\begin{cases} x(i,j,k) = [R + \Delta R(j,k)] sin\varphi(i,j,k) + x_t(i,k) - x_{tw}(i,k) + i\,\frac{N_f f_t}{N} \\ y(i,j,k) = [R + \Delta R(j,k)][1 - cos\varphi(i,j,k)] + y_t(i,k) - y_{tw}(i,k) \end{cases} \tag{11}$$

where N – number of iterations per revolution, R – cutter radius, $\Delta R(j,k)$ – cutter run out for *j*-th cutting edge and *k*-th level along the tool axis, N_f – number of teeth, φ – angular position, f_t – feed per tooth, x_t, y_t, x_{tw}, y_{tw} – displacements of tool and workpiece in x and y directions.

Another solution is based on storage of milling tool centers and cutting edge positions used for the generated surface calculation. The positions can be stored in the array as a function of rotation angle with the *j index* or simulation time intervals. Simulation of the milling process necessitates three-dimensional surface determination, which requires registration of three coordinate position of cutting edge (x, y, z), three coordinates of the cutter center (x_c, y_c, z_c) and angular position of the cutting edge (φ).

Storage of the tool positions history is not very memory consuming, but requires determination of outer modulation r_T for each iteration and for all segments of the cutting edge (eq. 8). The alternative is storage of the tool-workpiece displacements in radial direction r_t and use them in the next passes as r_T, which admittedly requires more computer memory but reduces the calculation cost.

The main non-linearity of the cutting process is tool jumping out of cut due to excessive vibrations, resulting in vanishing of cutting forces. It can be taken into account by more precise definition of the r_T value (outer modulation): it must be determined as a trace left on the surface generated in the previous or earlier passes (Fig. 6) [17],[18],[24],[34],[37]:

$$r_T = minimum\ \{\, r(t\text{-}T),\ h_0 + r(t\text{-}2T),\ 2h_0 + r(t\text{-}3T)\ldots \} \tag{12}$$

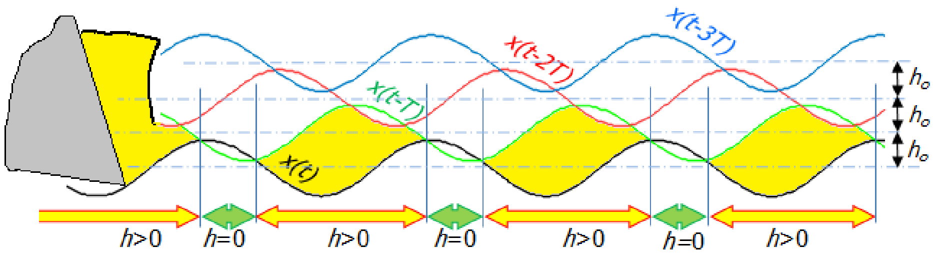

Fig. 6. Exact kinematics approach for surface updating: h>0 - tool in cut, regular cutting and h=0 tool out of cut due to excessive vibrations [18]

3.3. DYNAMIC FORCE DEPENDENCE ON THE UNCUT CHIP THICKNESS CHANGES

Fundamental importance for the stability analysis results has assumed model of the cutting process – the dependence of dynamic cutting forces on uncut chip thickness changes (stiffness components F_{rk}, F_{tk}) and velocity of vibrations in r direction (damping components F_{rc}, F_{tc}). These dependencies refer to the elementary segment of the cutting edge (for example 1mm), as shown in section 3.1, Fig. 4. The dependence of static (without vibration) forces on the constant uncut chip thickness h_0 determined experimentally can be expressed as non-linear function (Fig. 7a) [31]:

$$F_{j0} = C_j\, b\, h_0^a \quad \text{for } j=r, t, a \approx 0.7 \tag{13}$$

For the analytical stability limit determination these relationships can be linearized around steady state (Fig. 7b [14],[17],[30]). Thus stiffness components of the dynamic force are:

$$F_{jk} = bh_d\, \mathrm{d}F_{j0}/\mathrm{d}h\big|_{h=h_0} = bC_j a h_0^{a-1} h_d = bk_{jd}\, h_d \tag{14}$$

This locally linearized exponential model is easily applicable in turning, when the steady state uncut chip thickness h_0 is constant. The stability lobes calculated using such a model are feed rate dependent, which is a well-known effect, confirmed by practice. There are also more advanced models, which allow the identification of the dynamic cutting force coefficients via steady-state cutting tests e.g. that of Nigm et al [26]. The dynamic cutting force model presented in that of Jemielniak [15] redefines the uncut chip thickness, the cutting forces and the tool geometry (rake and clearance angle) with respect to the dynamic coordinate system, which is based on the instantaneous cutting speed v_e (see Fig. 1). The coefficients of this model can also be identified via steady-state cutting tests.

In milling uncut chip thickness is inherently variable, sometimes from zero to the maximum, so there is not steady state uncut chip thickness. Thus, the most common is Altintas model [3],[5], which is based on a simplified dependence of F_{j0} on h:

$$F_{j0} = k_{je} b + k_{jc} b h_0 \quad \text{for } j=r, t \tag{15}$$

The model consists of two components. The first is the force component related to the friction and ploughing (proportional to the length of the cutting edge *b*), given by the k_{je} coefficients. The second component is related to the material shearing (proportional to undeformed chip section bh_0), given by the k_{jc} coefficients. Only the second component of the force is influenced by changes of uncut chip thickness thus according to this model the stiffness components of the dynamic forces is [7] (Fig. 7c):

$$F_{jk} = k_{jc} b h_d \quad \text{for } j=r,\, t \tag{16}$$

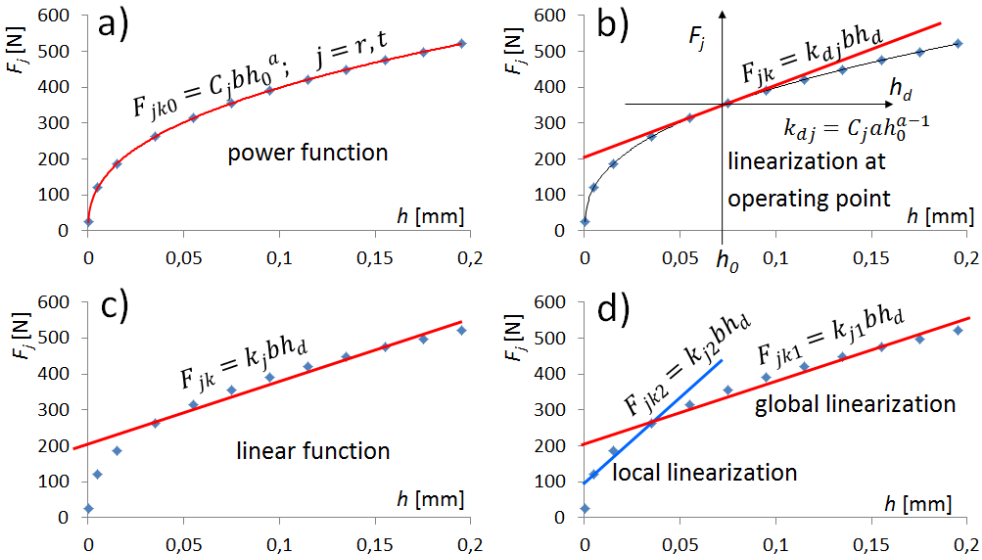

Fig. 7. Dependence of cutting force on the uncut chip thickness: a) non-linear power function, b) linearization at the operating point, c) linear function, d) local and global linearization

As can be seen in Fig. 7c, this model is very inaccurate at low uncut chip thickness. The stability limit calculated using this linear model in not feed rate dependent. Therefore sometimes set of local models are used [12],[32], covering the different cutting conditions, e.g. within a certain range of the uncut chip thickness (Fig. 7d). On the other side even simpler model of the dynamic forces is sometimes used, where the radial force is proportional to the tangential force [6],[7,[38]:

$$F_{tk} = k_{tc} b h_d; \quad F_{rk} = k_{rc} F_{tk} \tag{17}$$

where cutting coefficients k_{tc} and k_{rc} are constant

It should be strongly emphasized here that numerical simulation allows application of any characteristics of the dynamic cutting forces and accurate calculation of the force value at each iteration, for each point of the cutting edge and corresponding actual uncut chip thickness. Any non-linearity in the process such as varying cutting coefficients along the insert (equation 13), discontinuities along the flute due to inserts, and tool jumping out of cut due to excessive vibrations can be easily considered. The cutting force characteristic can be also used in tabular form based on experimental data. It is worth noting that there were many efforts devoted to the modeling of the actual cutting forces dependence on uncut chip thickness, e.g., those of Engin & Altintas, and Kaymakci [10],[19], and these results can be easily used for numerical simulation of self-excited vibration.

3.4. DAMPING OF THE CUTTING PROCESS

Since the earliest works the influence of the working clearance angle α_{oe} on the stability limit was noticed [20],[39]. Well-known low speed stability effect is caused by interference between the flank face of the cutting surface [20], resulting from the declining working clearance angle (α_{oe} Fig. 1), which occurs only for negative values of vibration velocity r_t'. Damping component of the cutting force, usually is assumed to be proportional to the inclination of effective cutting speed η which is proportional to the vibration velocity r_t', (Fig. 1, eq. 4) [1],[15],[35],[36]:

$$F_{jc} = -\frac{c_j\, r_t{'}}{v_c} b \tag{18}$$

where c_j – process damping coefficient. The value of the c_j is constant, averaged for the entire oscillation period, often determined by comparing the experimentally and analytically obtained stability limit [29]. Jemielniak and Widota [17] adopted additional assumption that the working clearance angle α_{oe} (see Fig. 1) during dynamic cutting is always positive. As a result, the process damping force increases to infinity as the instantaneous clearance angle approaches zero. This effect was achieved by using dependence describing the damping force in the form:

$$F_{jc} = \begin{cases} -c_j b\, r_t{'} & \text{for } \; r_t{'} \geq 0 \\ -c_j b\, r_t' + LSS \frac{r_t{'}/v_0}{\alpha_0 + r_t{'}/v_0} & \text{for } \; r_t{'} < 0 \end{cases} \qquad \text{for } j = r, t \tag{19}$$

where LSS – low speed stability coefficient determined experimentally.

On the other side since the early works [23],[41] some researchers assumed, that the working clearance angle α_{oe} can be negative, causing compression of the workpiece material. A volume of the work material is pressed by the tool flank when the tool travels in the downward direction. A resistance force is generated at this deformed work material against the further penetration of the tool into the work material and is proportional to the indented volume V. Thus the dumping force components are described as:

$$F_{rc} = k_{rc} V; \quad F_{tc} = \mu F_{rc} \tag{20}$$

where μ – the friction coefficient.

It should be noticed, the interference between the tool flank and machined surface does not occur where the slope of machined surface becomes positive [20]. Recently many attempts to model this phenomenon were undertaken by modeling of contact pressure and the volume of the deformed material under flank face [8],[33],[35]. However, even after such complex evaluation, the final results are averaged over the whole vibration cycle and simplified to one linear factor as in equation (18). Results presented in Fig. 8 shows the importance of process damping especially for low speeds.

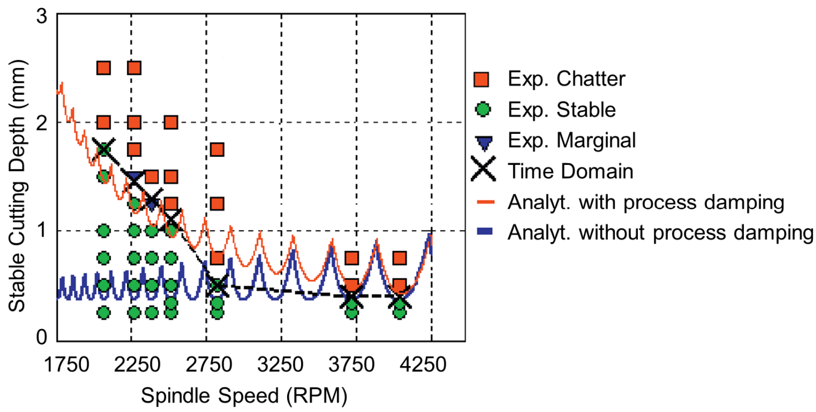

Fig. 8. Comparison of the stability limit obtained experimentally, by time domain simulation, with analytical solution with process damping and without it [8]

Again, using numerical simulation any nonlinearities of the process, including its damping can be easily taken into account. The damping force can be calculated for each iteration step separately, considering any complex dependencies of this force on instantaneous conditions.

Obviously this dependence can be replaced by any other, resulting from modeling or experimental results. Sometimes the influence of tool wear on process damping is considered [1],[2]. In that of Altintas et al [4], the influence of the flank–wave contact was modeled by considering both slope (r_t'/v_c) and curvature (r_t''/v_c^2) of the waves.

3.5. DETERMINATION OF RESULTANT CUTTING FORCES

The resultant cutting force is the sum of the forces acting on each small segment of cutting edge engaged in cutting process due to the regeneration and tool rotation (Fig. 3), for all teeth. Each elementary force is calculated on the base of the assumed cutting force model (see above) as a sum of stiffness and damping components of cutting force. In a specific angular position of the cutter φ_{ij} cutting force distribution changes along the cutting edge. Projection of forces acting on the edge segments in the process system (F_{tij}, F_{rij}) to the machine system (F_{xij}, F_{yij}) also changes along the edge with the angle φ_{ij}:

$$F_j = \sum_{k=1}^{z_n} \sum_{i=\varphi_1}^{\varphi_2} \sum_{j=1}^{a_p} (F_{jk} + F_{jc}) \quad \text{for } j = r, t \tag{21}$$

where i subsequent angular cutter position φ, k – index of the tooth, z_n – the teeth number, j – segment number along the axis.

Time domain numerical simulation methods allow the nonlinearities and various tool geometries, and – which is maybe most important – allow considering the cutting forces time inevitably varying with the uncut chip thickness changing in milling even without any vibrations. In frequency domain solutions the forces are integrated, averaged and considered constant in stable machining [3].

4. ACCURACY AND COMPUTING COST OF SIMULATION

Time domain simulations can consider any characteristic of cutting process and machine tool structure, the influence of inner and outer modulation (e.g. exact dynamic chip thickness history), tool geometry, run-out and other non-linearity such as tool jumping out of cut, and past multiple regeneration waviness caused by previous passes of the tool (previous teeth). Therefore, any frequency domain chatter stability solution should be compared against a numerical, time domain solution as a benchmark [7].

The time domain simulation is very time consuming due to necessity of calculating the high number of the cutting edge segment positions left on machined surface, which must be kept in the memory for accurate determination of the uncut chip thickness and multiple regeneration. Simulation accuracy depends strongly on the number of cutting edge segments inversely proportional to the segment size, especially when the number is small. The same is with the iteration time (step dt eq.4) – the shorter the better. Of course, computation time is proportional to number of cutting edge segments and number of iteration steps per second. However after crossing some number of segments and iteration steps the accuracy effectiveness of increasing these numbers becomes less and less eminent still causing the rise of computational costs [18]. Also shape of a round cutting edge segments has a strong influence on simulation accuracy and computational cost. For this reason division of the ball end-mill to equal segments along the tool axis seems to be not justified [22]. A significant computing cost reduction (simulation time), especially for the stability lobes determination, can be achieved by separation of triangular and quadrilateral segments [23],[35] or matching discretization steps according to the distance from the stability limit, determined by successive approximations [18]. Therefore it is recommended to take relatively large steps in the beginning (far from the designated stability limit), while decreasing near to the final solution. At the beginning of the century the stability lobe generation in the time domain took approximately two days of computer time and only a few seconds in the frequency domain on a personal computer [10]. Computational power increased several times since then, thus nowadays such computation takes several minutes.

5. SUMMARY AND CONCLUSIONS

Analytical prediction of the stability limit in frequency domain provides fast and easy obtainable results. However they are based on radically simplified, linear models of the cutting process and machine tool characteristics. It is impossible or very difficult to take into account these nonlinearities and – which is even more harmful – to consider continuous changes of these characteristics due to e.g. change of the workpiece shape. Numerical simulation of self-excited vibrations has not these limitations. Any characteristic of the cutting force or the machine tool system can be easily implemented, even in the form of direct experimental results, tables. The main limitation and drawback of the numerical simulation is a high computational cost (time of calculations). Therefore the algorithms and data structures in simulation program are very important and should be carefully prepared. On the other side, increasing computing power allows for prediction of greater use of numerical simulation especially for virtual machining. Integration of the machining process simulation enabling precise stability limit prediction directly to CAD/CAM systems will allow optimal planning of the machining operations in a virtual environment before testing them on costly trial cuts on the shop floor [7].

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