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COUPLED MODELLING FOR MACHINE TOOL STRUCTURAL OPTIMIZATION

Effective machine tool design needs to take into account various kinematic configurations and possible combinations of structural parts, which meet the requirements of both the structural properties on one hand and technology and cost limitation on the other hand. Prior to a detailed developing a certain machine tool structure, an expert based decision on the machine tool conception needs to be performed. A high number of design variants should be explored in a short time. Fulfilling those demands leads to developing a machine tool modular models, enabling easy changing the kinematic configuration or various structural parts. In the paper the techniques for effective component coupling and model order reduction using mode truncation or Krylov subspace based technique for creating the machine tool coupled models are introduced. Case studies considering real machine tool structures are given. High quality of Krylov subspace reduction technique in connection with multipoint constraint surface coupling is shown both in terms of dynamic properties of the reduced multibody model and a very low time demands at the same time.

1. INTRODUCTION

Development of a new machine tool is usually defined at the very beginning by a general specification of a maximum workpiece size. According to a mix of various functional criterion, such as static and dynamic stiffness, precision, machining performance, cost etc. a suitable machine tool structural design needs to be found. A number of different kinematic configurations and combinations of various parts and components may generally be considered. In fact, an appropriate choice of the structural arrangement in an early design stage decides on the future properties and features of the machine tool.

Nowadays, an experience and transfer of previous knowledge is usually applied when deciding on the new machine tool concept. However, this approach allows only for evaluating a limited number of possible variants. To enhance the level of decision making, various database or computational tools may be applied. In [1] a machine tool configurator

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including also auxiliary technical and economic calculations has been proposed. Technique of modular machine tool modelling and its application for optimization tasks is presented in [2]. Parametrical optimization using coupled finite element (FE) simulations is introduced in [3]. In [4], an approach for parametric optimization based on groups of machine tool structural types is proposed.

In this paper, techniques for machine tool multibody system modelling providing fast response calculations suited for modular structural optimization are introduced and tested. Models should enable evaluating a high number of possible kinematic configurations or application of various parts and components, which may be shared across more machine tool types in a company production portfolio. Linear FE models of compliant structural parts and a common simplified representation of linear rolling guideway carriages by linear spring elements is therefore considered with respect to low computational demands, required in optimization tasks.

Joint stiffness plays an important role in the overall machine tool stiffness, whereby the stiffness and damping is generally non-linear and depending on various parameters. An extensive overview of joint properties and behaviour is given in [5]. Back showed that a common joint representation by the spring or beam element is a reliable practice [6]. Yang et al. [7] expressed a mathematical model for a bolted joint by considering translational and rotational spring stiffness using experimental values. Böswald et al. [8] proposed an analytical model for joints by using a linear spring, nonlinear spring, and damping element. Impact of joint non-linear characteristics on the machine tool dynamic compliance and its relation to stable machining limits has been studied e.g. by Watanabe and Sato [9]. They proposed the so-called Nonlinear Building Block approach to obtain the frequency response of a structural system with nonlinear springs connecting linear components.

A simplified assumption of rigid joints is considered in this study as a basis for comparing the mathematical models of coupling the reduced FE models by spring elements or constraint equations. Interest is focusing on employing the mode truncation model order reduction (MOR) technique and its comparison with the Krylov subspace based reduction method, the application of which has been developed for creating a machine tool multi body model.

The first part of the paper introduces the component coupling methods followed by the description of model order reduction techniques chosen in the study. In the second part, case studies on comparing the results of component coupling methods and application of model reduction using an example of a real machine tool is given.

2. COMPONENT INTERFACE MODELLING

Machine tool structural components are usually joined to each other on interface surfaces by preload screw joints. A suitable simplified representation of surface joints needs to be found for structural optimization tasks which require quick computational response. The model has to be linear with respect to low computational demands and easy to implement into the machine tool multibody system (MBS).

Within this paper, two strategies for surface coupling in MBS simulations are discussed. The first one is based on creating rigid regions with master nodes connected to the interface surfaces and coupling the master nodes by spring elements. This strategy is very easy and convenient especially if finite element (FE) bodies are transformed and reduced into the first order state space system, which introduces force inputs and displacement outputs. A similar technique known as reacceptance coupling employing the spring elements for coupling the frequency transfer functions is commonly used e.g. for coupling a detailed spindle model with the ram structure, or tool coupling with spindle [10].

The other strategy couples directly surface to surface by means of linear multipoint constraint (MPC) equations, created between the FE nodes of each of the bodies. The MPC technique allows also for coupling of non-conform FE meshes and therefore does not require any special treatment of the FE models.

In this chapter, basis of force coupling in state space and general framework for coupling using the multipoint constraint equations is introduced.

2.1. COUPLING IN STATE SPACE

There are several ways to couple models in state space. The easiest to implement is the technique based on force coupling by means of spring elements. General scheme of this approach is given in Fig. 1.

System of State-Space equations is written as:

$$\begin{aligned} \dot{x} &= \boldsymbol{A} \cdot x + \boldsymbol{B} \cdot u \\ y &= \boldsymbol{C} \cdot x + \boldsymbol{D} \cdot u \end{aligned} \tag{1}$$

where: x is the state vector of the system, u the vector of forces and y the output vector. Matrixes $\boldsymbol{A}$ and $\boldsymbol{B}$ are the input matrices, matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ are matrixes of the system output.

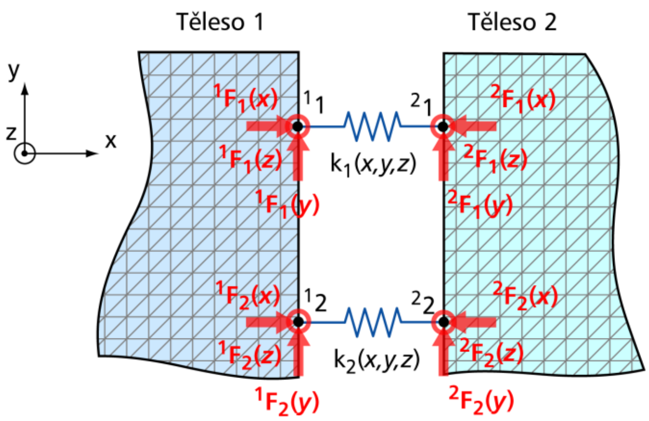

Fig. 1. Coupling of two FE models with springs

The equilibrium equation in the connected degrees of freedom is expressed as:

$$-\,{}^{1}F_i = {}^{2}F_i = k_i\left({}^{1}x_i - {}^{2}x_i\right) \tag{2}$$

State space equations of the coupled system can then be written as:

$$\begin{bmatrix} {}^{1}\dot{q}_s \\ {}^{2}\dot{q}_s \end{bmatrix} = \begin{bmatrix} {}^{1}\boldsymbol{A} & 0 \\ 0 & {}^{2}\boldsymbol{A} \end{bmatrix} \begin{bmatrix} {}^{1}q_s \\ {}^{2}q_s \end{bmatrix} + \begin{bmatrix} {}^{1}\boldsymbol{B} & 0 \\ 0 & {}^{2}\boldsymbol{B} \end{bmatrix} \begin{bmatrix} {}^{1}u \\ {}^{2}u \end{bmatrix} \tag{3}$$

Considering the transformation of modal to physical coordinates $x = \boldsymbol{V} \cdot q$, where $\boldsymbol{V}$ is a matrix of eigenvectors and q a vector of modal coordinates, the equation (2) can be rearranged as:

$$-\,{}^{1}u_c = {}^{2}u_c = k\left({}^{1}\boldsymbol{V}_c\ {}^{1}q - {}^{2}\boldsymbol{V}_c\ {}^{2}q\right) \tag{4}$$

Substituting (4) to (3) yields

$$\begin{bmatrix} {}^{1}\dot{q}_s \\ {}^{1}\dot{q}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{c1} & \boldsymbol{A}_{c2} \\ \boldsymbol{A}_{c3} & \boldsymbol{A}_{c4} \end{bmatrix} \begin{bmatrix} {}^{1}q_s \\ {}^{2}q_s \end{bmatrix} + \begin{bmatrix} {}^{1}\boldsymbol{B}_r & 0 \\ 0 & {}^{2}\boldsymbol{B}_r \end{bmatrix} u_{(r_1+r_2)} \tag{5}$$

which represents a general form of the state space input equation of a two bodies coupled system, described by $\boldsymbol{A}_c$ and $\boldsymbol{B}_c$ matrices. Similarly, the system output equation with the $\boldsymbol{C}_c$ and $\boldsymbol{D}_c$ matrices can be derived.

2.2. GENERAL FRAMEWORK FOR DYNAMIC SUBSTRUCTURING

Another way to couple dynamic systems is to connect their mass $\boldsymbol{M}_s$ damping $\boldsymbol{C}_s$ and stiffness $\boldsymbol{K}_s$ matrices, external force vectors f_s and internal force vectors g_s by coupling equations [11], whereby the subscript s denotes the systems being coupled. This approach is similar to global matrix assembly in FE software. The equations of motion of n coupled subsystems can be written as:

$$\boldsymbol{M}\ddot{x} + \boldsymbol{C}\dot{x} + \boldsymbol{K}x = f + g \tag{6}$$

The matrices $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are diagonal matrices containing the subsystem matrices. Vectors f and g are column vectors containing the subsystem internal and external force vectors respectively.

$$\begin{aligned} \boldsymbol{M} &= diag(\boldsymbol{M}^{(1)}, \dots, \boldsymbol{M}^{(n)}) \\ \boldsymbol{C} &= diag\left(\boldsymbol{C}^{(1)}, \dots, \boldsymbol{C}^{(n)}\right) \\ \boldsymbol{K} &= diag(\boldsymbol{K}^{(1)}, \dots, \boldsymbol{K}^{(n)}) \end{aligned} \qquad x = \begin{bmatrix} x^{(1)} \\ \dots \\ x^{(n)} \end{bmatrix}\ f = \begin{bmatrix} f^{(1)} \\ \dots \\ f^{(n)} \end{bmatrix} g = \begin{bmatrix} g^{(1)} \\ \dots \\ g^{(n)} \end{bmatrix} \tag{7}$$

Compatibility condition (coupling equations) is expressed as:

$$\boldsymbol{B}x = 0 \tag{8}$$

where: $\boldsymbol{B}$ is Boolean if the interface degrees of freedom match (interfaces are conforming). In this case the coupling equations are very simple $x^{(k)} - x^{(l)} = 0$. If the connected degrees of freedom don't coincide, the relations among them are more complex and the matrix $\boldsymbol{B}$ is a real matrix.

Equilibrium condition is given by:

$$\boldsymbol{L}^T g = 0, \tag{9}$$

where: $\boldsymbol{L}$ is Boolean matrix describing the relations among the interface forces. $\boldsymbol{L}$ is null space of $\boldsymbol{B}$.

The matrices of coupled system $\widetilde{\boldsymbol{M}}, \widetilde{\boldsymbol{C}}, \widetilde{\boldsymbol{K}}$ and vector $\tilde{f}$ are expressed as

$$\begin{aligned} \widetilde{\boldsymbol{M}} &\triangleq \boldsymbol{L}^{\boldsymbol{T}}\boldsymbol{M}\boldsymbol{L} \\ \widetilde{\boldsymbol{C}} &\triangleq \boldsymbol{L}^T\boldsymbol{C}\boldsymbol{L} \\ \widetilde{\boldsymbol{K}} &\triangleq \boldsymbol{L}^T\boldsymbol{K}\boldsymbol{L} \\ \tilde{f} &\triangleq \boldsymbol{L}^T f \end{aligned} \tag{10}$$

This approach allows coupling of full or reduced models in physical domain as well as in frequency domain (FRF coupling).

3. MODEL ORDER REDUCTION METHODS OF SECOND ORDER SYSTEMS

Accelerating the MBS simulations and making them feasible in practice requires model order reduction techniques (MOR) of the FE bodies. The idea behind MOR is to reduce the number of unknowns while producing sufficiently good approximation to the input/output behaviour. In this paper, mode truncation and Krylov subspace reduction techniques are introduced.

3.1. MODE TRUNCATION

One of the less complex methods of model order reduction is modal truncation method [12]. The projection matrix $\boldsymbol{V}$ is in this case obtained from modal analysis performed on the full model.

The equation of motion of the full model is:

$$\boldsymbol{M}\ddot{x} + \boldsymbol{C}\dot{x} + \boldsymbol{K}x = f \tag{11}$$

where the x is the displacement vector.

The projection matrix $\boldsymbol{V}$ consists of the system's mode shapes; each column of $\boldsymbol{V}$ corresponds to one mode shape $\boldsymbol{v_i}$ and each row represents a degree of freedom:

$$\boldsymbol{V} = [v_1 \ \dots v_n] \tag{12}$$

The vector x can be transformed to vector of modal coordinates using equation:

$$x = \boldsymbol{V}q \tag{13}$$

This equation can be substituted to (11) and after multiplying the result by $\boldsymbol{V}^T$ from the left hand side, one obtains the equation of motion of the full system in modal coordinates.

$$\boldsymbol{E}\ddot{q} + \boldsymbol{C}_q\dot{q} + \boldsymbol{\Lambda}q = \boldsymbol{V}^T f, \tag{14}$$

where: $\boldsymbol{E}$ is identity matrix, $\boldsymbol{C}_q$ is damping ratio matrix and $\boldsymbol{\Lambda}$ is matrix of eigenvalues. The equations in (14) are independent because the matrices $\boldsymbol{E}, \boldsymbol{C}_q,$ and $\boldsymbol{\Lambda}$ are diagonal. This means that the whole system of equations can be look at as sum of many one degree of freedom systems (Mode Superposition).

Disadvantage of this method is the fact that the reduced model's FRF never fits the full model's FRF near 0Hz. Another drawback is relatively long time needed to create the reduced model (modal analysis with extraction of lots of modes has to be performed). The method is on the other hand simple and accurate in chosen frequency range.

3.2. KRYLOV MOR

In this section only the basics behind Krylov reductions is described. The reader is encouraged to read an excellent mathematical description of Krylov based reductions in [13]. An overview of reduction methods is given in [14]. Although optimal Krylov based reduction algorithms are available [15] a simpler and possibly more computational efficient method is used in this work - a block rational Arnoldi method [16]. The structure of second order ODEs in (11) is preserved using Bai's algorithm [17].

The Laplace transform of eq. (11) has the form of:

$$\boldsymbol{H}(s) = \boldsymbol{L}^T(s^2\boldsymbol{M} + s\boldsymbol{D} + \boldsymbol{K})^{-1}\boldsymbol{F}. \tag{15}$$

And the McLaurin series of transfer function (15) has the form:

$$\boldsymbol{H}(s) = \sum_{i=0}^{\infty} m_l s^l, \tag{16}$$

where: m_l are the so-called moments of the transfer function:

$$m_l = L^T r_l, \tag{17}$$

$$\begin{gathered} r_0 = \boldsymbol{K}^{-1}\boldsymbol{F} \\ r_1 = -\boldsymbol{K}^{-1}\boldsymbol{D}r_0 \\ r_l = -\boldsymbol{K}^{-1}(\boldsymbol{D}r_{l-1} - \boldsymbol{M}r_{l-2}). \end{gathered} \tag{18}$$

The first *n* vectors r_l span Krylov space:

$$\boldsymbol{K}_n = span(r_0, \cdots, r_{n-1}). \tag{19}$$

Let V_n be the orthonormal basis of K_n:

$$\boldsymbol{K}_n = span(V_n), \qquad V_n^T V_n = I, \quad V_n \in R^{Nxn}. \tag{20}$$

The projection of state coordinates q onto $\boldsymbol{K}_n$ using V_n is called generalized state coordinates $q \in R^n$:

$$x = \boldsymbol{V}q + \epsilon. \tag{21}$$

The error $\epsilon \in R^n$ in the projection rises while performing projection of *x* onto K_n.

We obtain reduced system of eq. (22) by substituting generalized coordinates q into equation (11) and using Galerkin method. The reduced equations have the form of:

$$\begin{gathered} \boldsymbol{M}_n \ddot{q}(t) + \boldsymbol{C}_n \dot{q}(t) + \boldsymbol{K}_n q(t) = \boldsymbol{F}_n u(t) \\ \tilde{y}(t) = \boldsymbol{L}_n^T q(t), \end{gathered} \tag{22}$$

where

$$\begin{gathered} \boldsymbol{M}_n = V_n^T \boldsymbol{M} V_n \\ \boldsymbol{K}_n = V_n^T \boldsymbol{K} V_n \\ \boldsymbol{C}_n = V_n^T \boldsymbol{C} V_n \\ \boldsymbol{F}_n = V_n^T \boldsymbol{F}, \end{gathered} \tag{23}$$

where $\boldsymbol{M}_n, \boldsymbol{C}_n, \boldsymbol{K}_n \in R^{nxn}, \boldsymbol{F} \in R^n, \boldsymbol{L}_n \in R^{nxm}$.

The transfer function of the reduced system (23) has the form:

$$\boldsymbol{H}_n(s) = \boldsymbol{L}_n^T(s\boldsymbol{C}_n + \boldsymbol{K}_n)^{-1}\boldsymbol{Q}_n. \tag{24}$$

The above procedure assures that the first n moments of the transfer function (15) of the full system equals to the first n moments of the transfer function (24) of the reduced system [13].

The procedure is easily extended to a multi-input/multi-output case where $\boldsymbol{Q}, \boldsymbol{F} \in R^{Nxl}$ and $\boldsymbol{L} \in R^{Nxm}$. The size of the reduced system is determined by the size of $\boldsymbol{Q}$ and $\boldsymbol{L}$. However it is possible to use the superposition property [18] to keep the matrices small.

4. COMPARISON OF COUPLING TECHNIQUES

A case study on comparing the coupling techniques with respect to the structural properties of a multibody model is performed using a simplified ram and spindle model (Fig. 2). Spindle and ram are connected to each other at the front of the ram and at one of the internal ribs, supporting the back of the spindle. Directional frequency response functions (FRFs) at the tool tip (reference point) are evaluated. FE models of both the ram and spindle are reduced using the mode truncation technique, whereby first 100 eigenmodes are retained.

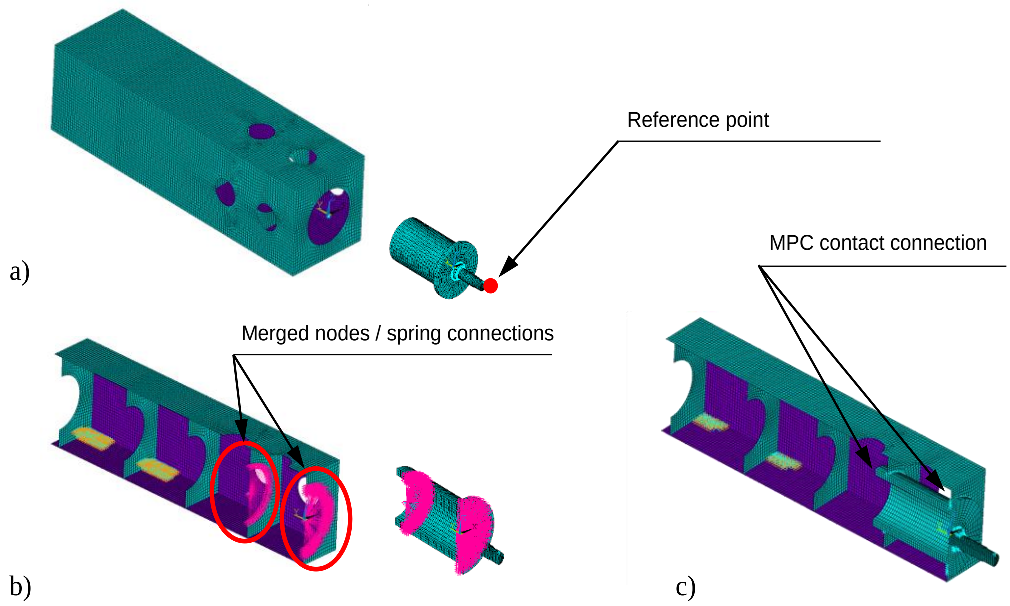

Fig. 2. Ram and spindle case study coupled model: a) models to be connected,b) spiders prepared for merging or coupling master nodes with springs, c) MPC contact connection

The first case considers coupling in state space by employing a force interaction between the master nodes, connected to the interface surfaces by nets of constraint equations (magenta lines in Fig. 2b). Three springs with longitudinal degrees of freedom (DOF), or six springs in all of the DOFs are created between the master nodes. The other case, which is considered to be a reference case in the study, introduces MPC contacts between the interface surfaces (Fig. 2c).

A question of an appropriate setting of both the translational and torsional spring stiffness used for coupling the bodies is studied. Translational spring stiffness is selected in the range of 10^{12}–10^{16} N/m, the stiffness of torsional springs in the range of 10^{9}–10^{12} Nm/rad. The values used are considered to represent an absolute stiffness level.

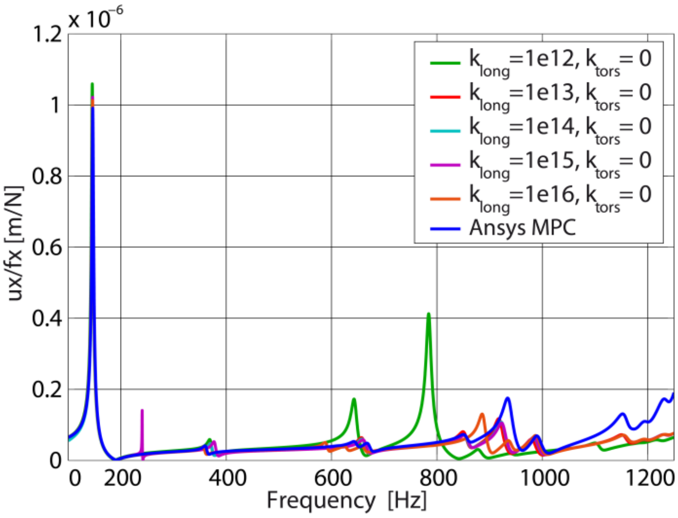

Fig. 3. Coupled model behaviour for different stiffness of translational coupling springs [N/m]

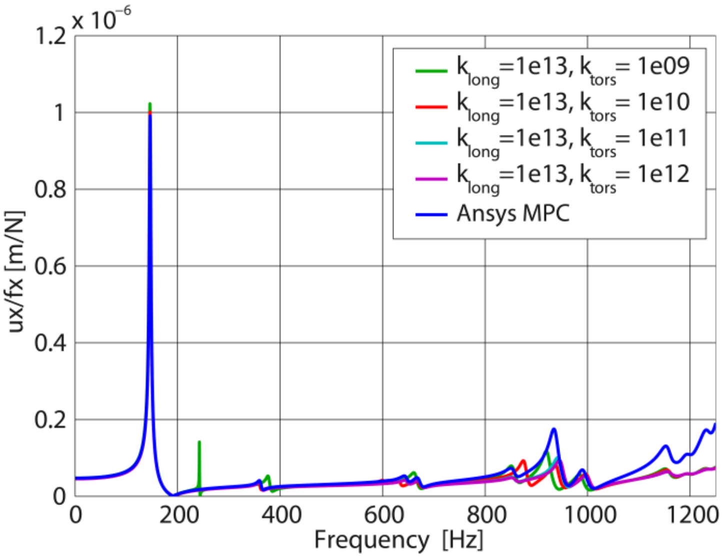

Fig. 4. Coupled model behaviour for different stiffness of torsional coupling springs (Nm/rad) and translational stiffness of 10^{13} N/m

A very good match of the spring coupled model with the reference case represented by the full FE model using MPC contact connection is found for the first eigenfrequency regardless of the spring stiffness. However, quite significant shifts in FRFs are observed in higher frequency range (800–1000Hz). In Fig. 3 the impact of translational spring stiffness is shown, the Fig. 4 presents the effect of the torsional springs; in both cases the reference model is depicted by a blue curve. The results reveal that modelling the surface coupling by spring elements between the master nodes is very sensitive to the spring stiffness. Therefore, it becomes obvious, that this approach does not represent a technique robust enough for surface coupling.

5. CASE STUDY OF MACHINE TOOL MULTIBODY MODEL

The purpose of the study is to perform a mathematical verification of the FRFs generated at the TCP of a large portal milling machine tool. Besides applying the multipoint constraint (MPC) equations for component coupling, the study aims also at testing the suitability of reduction methods for creating a machine tool multibody model, assembled from reduced FE models of separate structural parts. Modal reduction technique and Krylov subspace method are considered. In this way, a model allowing for quick update of machine tool dynamic properties according to actual kinematic configuration is created.

View of the whole structure gives Fig. 5. The FE model is composed of volume, shell, spring, matrix and mass elements. The mesh consists of about $1{,}5.10^6$ nodes, the total number of degrees of freedom number is almost 6.10^6. The model was built in ANSYS v14.5.

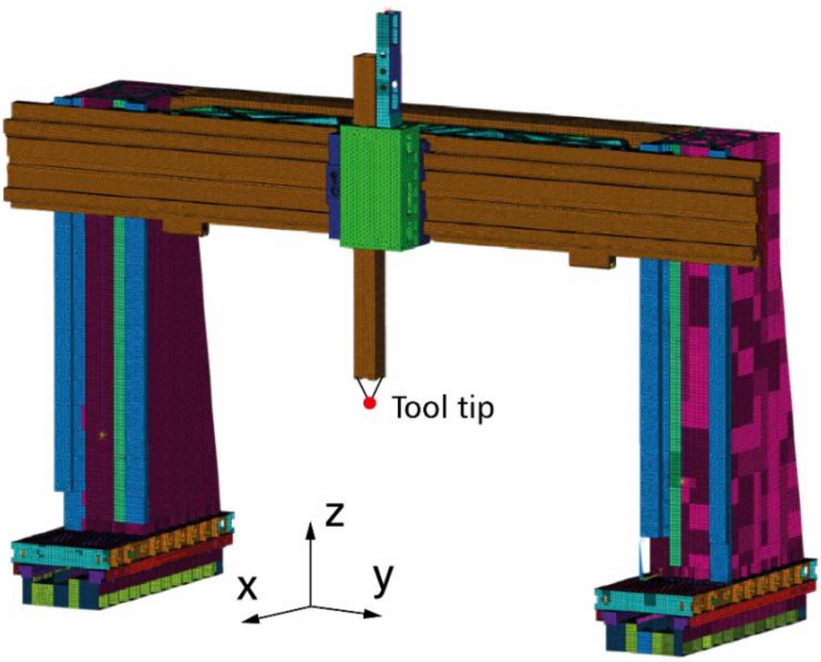

Fig. 5 Machine tool FE model

Model decomposed into the separate structural parts introduces Fig. 6 with a description of type of coupling chosen according to the character of connections between the parts. Surface joints between the structural parts (*X*-slide, column, fixed cross beam) are modelled using the TCP linear contacts (*face coupling*). Connection of movable parts, such as *X*-slide to base or *Y*-slide to movable cross beam is realized by linear spring elements (symbols of springs in Fig. 6) which represent linear motion guideway carriages or hydrostatic pockets. Rack and pinion or ball screw feed drives are modelled using linear spring elements, which reflect a closed position control.

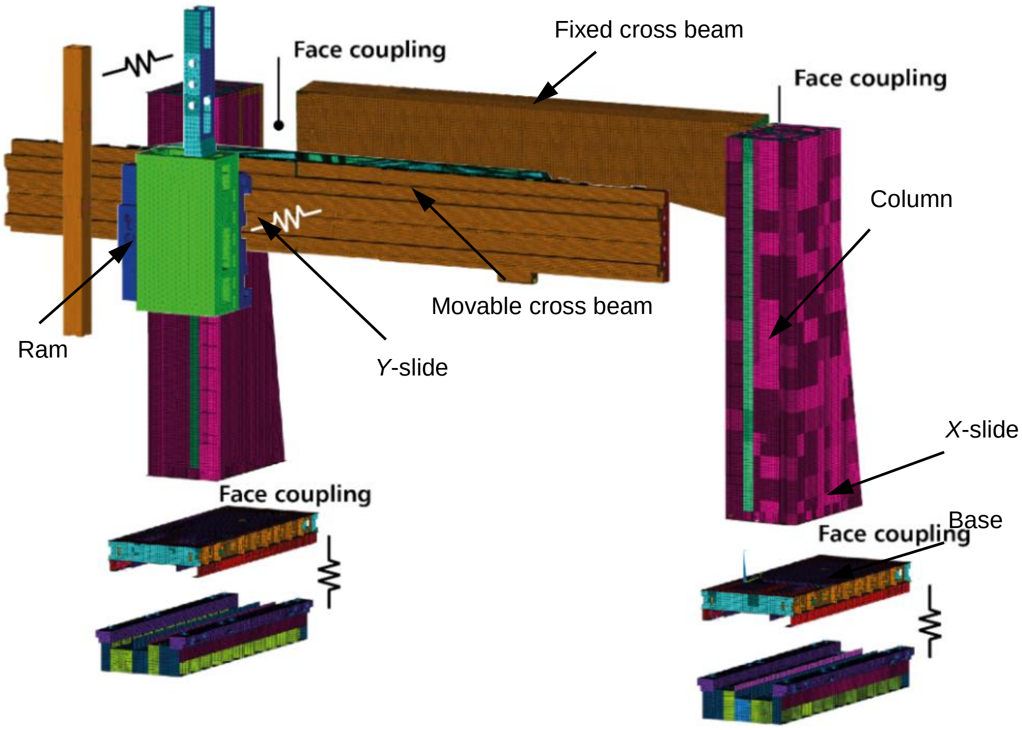

Fig. 6. Multibody system of the machine tool structure

Table 1 shows comparison of methods in terms of time needed to produce the reduced order model (MOR), time to compute the FRF and the total time (MOR+FRF). It may be seen that by using the Krylov MOR, the total time needed for obtaining a required FRF is almost approx. 1000 x shorter then by using the full harmonic analysis, or approx. 5.8 x shorter compared to the mode truncation method. Next to it, significant advantage of employing the reduced models is also that they are computed just once and can consequently be coupled in different positions, so allowing a quick analysis of machine tool behaviour in the whole working space.

The Fig. 7 and Fig. 8 show FRFs in *X* and *Y* axis. The results clearly show the quality of approximation obtained using Krylov MOR, the error of which is negligible. The mode truncation is computationally less efficient and also the approximation error on frequencies higher than 50Hz is very high rendering this method almost unusable.

Table 1. Comparison of computational times

	MOR	FRF simulation	Total time
Full harmonic	-	111hours	111hours
Mode truncation	40min	< 1s	40min
Krylov MOR	6.9min	< 1s	6.9min

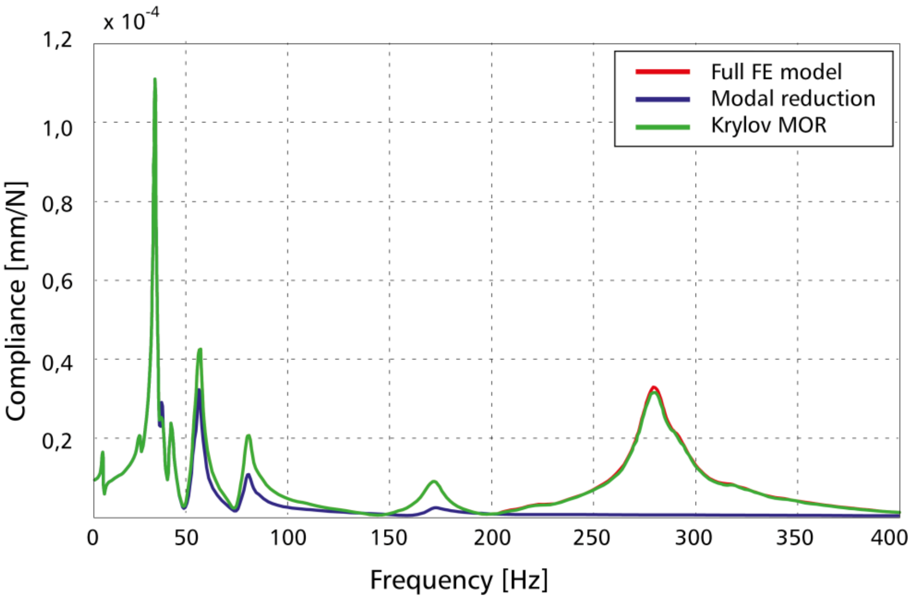

Fig. 7. Comparison of FRF, axis *X*

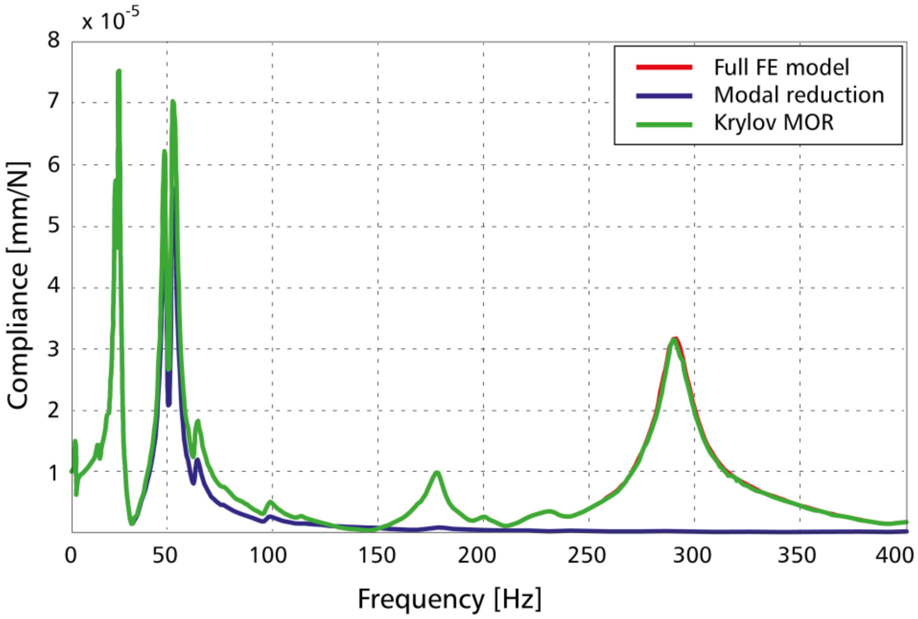

Fig. 8. Comparison of FRF, axis *Y*

6. CONCLUSIONS

In the paper a study on component coupling techniques for machine tool multibody modelling has been presented and a novel application of Krylov subspace based model order reduction method for coupling of FE bodies has been proposed and successfully tested. The application of the techniques studied aims at machine tool multibody modelling for modular optimization tasks.

It has been shown that force interaction based surface coupling method using spring elements does not provide reliable results with respect to the spring stiffness setting. Even in the range of stiffness values considered to represent an absolute stiffness level the properties of the coupled model may vary significantly.

To the best authors knowledge coupling of arbitrary FE substructures reduced using Krylov MOR has not yet been reported in literature. The proposed method has been tested on dynamic simulation of multibody system of a machine tool structure. The case study has shown superior quality of Krylov subspace base MOR in comparison to mode truncation technique for creating the machine tool multibody model, in which each of the structural parts is reduced separately and coupled to each other using either multipoint constraint (MPC), or force coupling. The multibody model employing the Krylov subspace base MOR features almost identical match of the dynamic properties with the full FE model. At the same time, Krylov subspace method is very time efficient and proves to provide a very effective tool for quick model updates related to varying structural dynamics with respect to various kinematic configurations or testing of different design variants in the machine tool development.

Next, an ongoing research on the interface surface properties in terms of stiffness and damping will help increase the quality of predicting the machine tool dynamic properties in an early design stage. Model enhanced by stiffness and damping characteristics of the interface contacts will enable more relevant predictions of directional dynamic compliancy at the tool and consequently a more effective evaluation of the structural optimization tasks according to complex criterion of workpiece quality and precision using a machine tool virtual model.

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