

*chatter, stability inaccuracy,
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A METHOD OF IDENTIFICATION OF COMPLEX CUTTING FORCES ACTING IN UNSTABLE CUTTING PROCESS

The calculations of machining stability limit currently known do not provide a precise prediction of chatter-free cutting conditions. The inaccuracy is probably caused by mathematical models of dynamic forces acting on the cutting process during unstable machining. These models need to be modified. A new analysis of experimental data measured by one of the authors, M. Poláček, in the 1968-1974 period, forecasted an existence of several dynamic forces, which are mutually phase-shifted, thus complex. This fact has not been thoroughly investigated previously. As the assumed forces cannot be calculated from any equations, they must be identified experimentally. This research paper proposes a theoretical method of an experimental identification of these forces. The new model is intended to be used in the future for the development of a more accurate calculation of stability diagram.

1. INTRODUCTION

Self-excited vibrations arise under certain conditions when machining. The oscillations arise when a flexible vibrating system machine-tool-workpiece deflects from its equilibrium position by a random impulse, for example by the cutting tool running up into the cut. Subsequently, the oscillating tool produces wavy surface. The surface waves and tool oscillations have the same period and they are phase-shifted. The result is the emergence of a periodically variable component of the cutting forces, which in a feedback loop also acts on the development of self-excited vibrations and produces oscillations with increasing amplitude. In a short period of time the amplitude of the oscillations stabilizes at a certain level.

There have been many attempts to calculate the stability limit of such an oscillating system. The first applicable calculation was developed in the 1950s for turning [10] and later published in [4]. According to Poláček's theory, the linear model of the dynamic components of the cutting forces was directly dependent on the variable thickness of the chip. The constant of proportionality was static specific cutting force also called specific

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cutting resistance. Width of chip was chosen as a measure of the stability limit. The linear model of dynamic forces led to a simple condition for the limit width of chip on the stability boundary. The relationship is successfully used today to solve the problems of instability of machine tool structure. In this case, the dominant role is played by the modal analysis of the machine structure. The linear model is, however, not completely suitable for the prediction of chatter-free cutting conditions. Deviations of the calculation are the largest in the area of lower cutting speed, which is today a major shortcoming in the machining of hard-to-machine materials such as titanium and nickel alloys [20].

Even in the area of higher cutting speed the accuracy of the prediction of stable cutting conditions is not, in some cases, satisfactory. Attempts to refine the prediction have been trying to remove these shortcomings [1]. However, the models of dynamic cutting forces used still hold to the premise of the original linear model, i.e. that when unstable machining comes into existence, one dynamic component of cutting forces is generated as a result of changing the thickness of the chips periodically.

A significant attempt to improve the stability prediction was the cooperative research of dynamic cutting force coefficients reviewed in [18] and [9]. In this investigation only one dynamic cutting force was considered. The force was decomposed into two components, a so-called direct component and a cross component. Both components consisted of the two forces generated by so-called inner and outer modulation. The research showed that the forces are complex and their phases depend on cutting speed. Unfortunately, the results of measurements of dynamic complex coefficients and their corresponding forces operating during unstable machining suffered considerable variance of measured data. According to the authors, it was due to insufficient monitoring of tool wear. Repeated measurements, published in [6],[17] and [19], confirmed this assumption and reduced the variance to some extent.

Although all these works confirmed the complexity of the forces, no conclusions regarding stability calculation were drawn from this fact, except a very simplified stability calculation. Under very specific conditions specified in [18], it is possible to deduce the following simplified relationship for the stability limit:

$$b_{\text{lim}} = \frac{4k\xi}{\{\text{Re}(K_{di}) + \text{Re}(K_{ci}) - \text{Im}(K_{di}) - \text{Im}(K_{ci})\}} \quad (1)$$

where: k and ξ are static stiffness and damping ratio of the single degree of freedom vibration system, $\text{Re}(K_{di})$ and $\text{Re}(K_{ci})$ are the real parts of dynamic cutting force coefficients (DCFC) for the direct and cross force. The direct and cross force correspond to F^a and F^b used in this paper. $\text{Im}(K_{di})$ and $\text{Im}(K_{ci})$ are the imaginary parts of the same coefficients (Fig. 1).

Dependences of measured coefficient components of inner modulation on cutting speed, as presented by Tlustý [18], are shown in Fig.1. Using these data, and for certain selected values k and ξ (these values do not influence the curve shape), we are able to calculate the dependence of the stability limit on cutting speed in Fig. 2., which is the bowl-shaped curve. This example is very specific and cannot be generalised for other cases, but the importance of it lies in the specific shape of the stability curve.

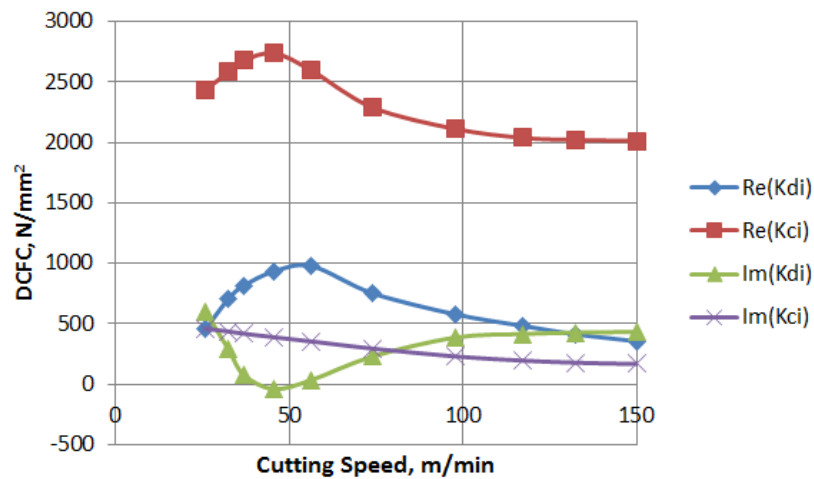


Fig. 1. Real and imaginary components of dynamic cutting force coefficients for turning. Data compiled from [18] and [9] and averaged. Material: steel 1045

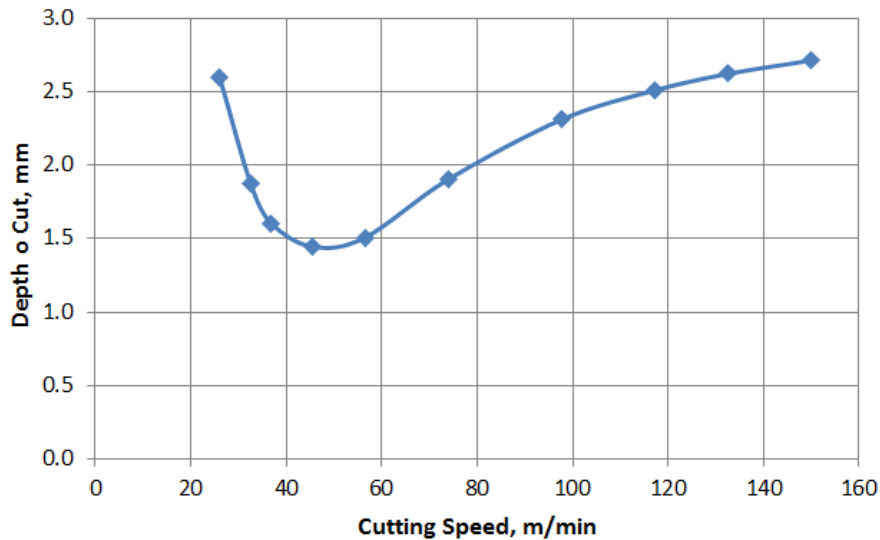


Fig. 2. Limit of stability for turning at lower cutting speeds calculated with the use of data published in [18] and [9]

The same bowl-shape was measured when turning and published in [5],[7] and [18] as well as in other works reviewed in [2] (see Fig. 3). The reason for the bowl-shaped stability curve has not been investigated until now but there is certainly a relationship to the dependence of complex dynamic cutting forces on cutting speed. There is also a great influence of tool geometry which was investigated in [8] and in [2]. Based on the above-mentioned review, we can say and deduce that:

- until now it has not been proved to satisfaction how the shape of a stability diagram can be influenced by the complex dynamic forces,
- as the dynamic forces are complex, they must be considered as individual forces, not as the geometrical components of one dynamic force, and must be treated only within a complex (Gauss) plain,
- these forces act at a cutting tool edge,

- d) each such a force has its magnitude, direction and phase shift, relative to a periodic deflection of the tool,
- e) each force causes a periodic deflection of a considered vibrating system, which participates in a resulting stability limit.

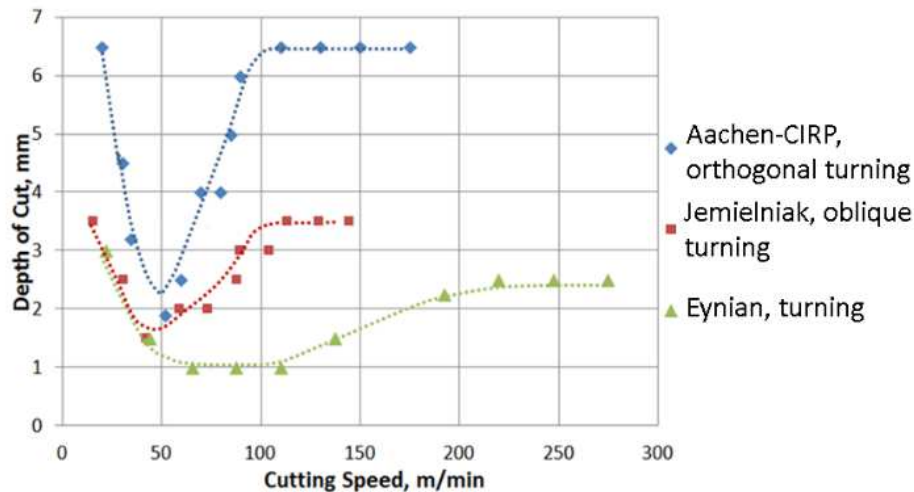


Fig. 3. Stability curves. Data compiled after [9],[7] and [5]

This paper firstly presents results of a new analysis of older data measured by one of the authors. Secondly, it suggests a new model of cutting forces, including a theory of their experimental identification. Finally, a prepared experiment is described briefly.

2. NATURE OF THE PROBLEM

The paper focuses on a method of identification of dynamic complex forces acting between the tool and the workpiece under unstable orthogonal machining using a single-edge cutting tool. The data obtained from the former tests published in [6],[11],[12],[14],[15],[16],[17] and [19], have been newly treated. The analysis has shown that there are very probably several dynamic forces of various sizes, orientations and phase shifting, i.e. complex forces, acting on the cutting edge, [2]. These forces are dependent on the parameters of the cutting process. An analytical solution of this task is not possible due to the complexity of this case. Therefore, an experimental identification of the forces will be applied. On this basis, a new model of dynamic forces will be formulated, taking into account their complex character. It is assumed that this model will refine the calculation of the stability limit. The accuracy of the stability limit calculation depends on conformity of the force model and natural reality existing during unstable machining.

The complexity of the task lies in the fact that the forces are generally complex forces of different sizes, directions and phases. *Individual forces cannot be directly measured. It is, however, possible to measure the summation of the individual forces projected in two*

mutually perpendicular directions. The projected summation is called the total forces in the text of the paper. The total forces are later decomposed into the projections of individual forces using a special method described below. The assumed dynamic forces will be determined experimentally in unstable machining. Unstable cutting conditions will be simulated by machining using a periodically excited cutting tool. A test device has been designed so that the tool and its flexible fastening to the machine form a single vibrating system with one degree of freedom. The mechanism of the machine on the side of the workpiece must be stiff enough to suppress machine vibration, including torsional vibration of a spindle drive.

3. METHOD OF SOLUTION

The method of the dynamic force identification is based on Poláček's previous work [13]. But the actual method considers some new findings of the authors obtained from the cited research reports of VÚOSO. As stated above, several dynamic forces, generally oblique, of different directions, sizes and phases operate during unstable machining. Generally, the vectors of the forces are three-dimensional, but the task will be simplified to a planar task. For this reason, the tool will cut a thin-walled tube in the parallel direction to the axis of the tube rotation (face turning). The width of the tool insert will be larger than the thickness of the tube wall. Thus the machining will be orthogonal without any radial force. We specify the total forces as F^a and F^b , where indices "a" and "b" denote the mutually orthogonal coordinate axes. The "a" axis is perpendicular to the direction of the axis of the machined surface. The direction of the "b" axis is the same as the direction of the cutting speed. See Fig. 4.

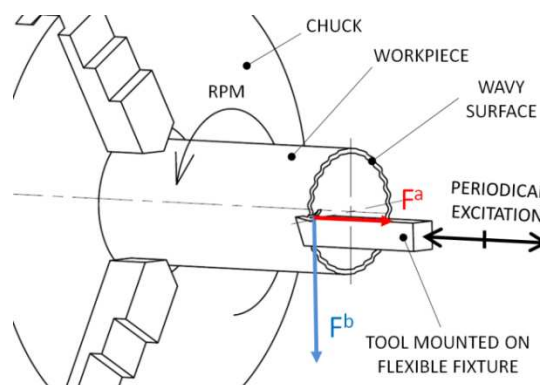


Fig. 4. The schema of the workpiece, tool and total force F^a and F^b

Each of the two forces F^a and F^b is given by the sum of the four components of the phasors F^a_i , F^a_o , F^a_D and F^a_w , and the phasors F^b_i , F^b_o , F^b_D and F^b_w , which are the projections of the dynamic forces in the direction of axes "a" and "b". These projections of the forces generally have a different phase.

$$\begin{aligned}
F^a &= F_i^a + F_o^a + F_D^a + F_W^a \\
F^b &= F_i^b + F_o^b + F_D^b + F_W^b
\end{aligned}
\tag{2}$$

Furthermore, we define the individual forces closer and explain the proposed method of their identification. For simplicity, we omit the indexes „a“ and „b“. The method of identification for both directions "a" and "b" is the same.

The first force F_i is a periodic cutting force. The index "i" refers to „inner modulation“, which means that the cutting force is generated in machining with the oscillating tool, whose cutting edge is located below the surface of the workpiece material (thus inner modulation). The second cutting force F_o is also periodic one. The index "o" stands for "outer modulation", which means that the cutting force originates in the removing of surface waves on the workpiece surface (thus outer modulation). Defining the force F_D , we accept at this moment the hypothesis of „process damping“ for the force. The force F_W is the damping force that depends on the wear of tool flank. Vectors of all the mentioned forces can be depicted in the complex plane along with the vector of the tool vibration Y_i , which lies on the real axis. The vector of surface waves Y_o is phase-shifted against Y_i by a phase of ε . See Fig. 5. The following relationship applies:

$$\begin{aligned}
Y_o &= Y_i \cdot e^{j\varepsilon} \\
|Y_o| &= |Y_i|
\end{aligned}
\tag{3}$$

where the stability limit is defined by the equality of amplitudes of the waves and tool oscillations. The indexes "i" and "o" indicate once again the "inner" and "outer" modulation. The phase ε can vary in the range of 0° to 360° . The reference signal for phase measurement of forces and waves is the oscillation of the tool Y_i .

Mathematical model suggests the forces F_i and F_o in the form:

$$\begin{aligned}
F_i &= K_i \cdot e^{j\alpha} \cdot b \cdot Y_i \\
F_o &= K_o \cdot e^{j\beta} \cdot b \cdot Y_o = K_o \cdot e^{j\beta} \cdot b \cdot Y_i \cdot e^{j\varepsilon} = K_o \cdot e^{j(\beta+\varepsilon)} \cdot b \cdot Y_i
\end{aligned}
\tag{4}$$

In the relationships (4) the expressions $K_i \cdot e^{j\alpha}$ and $K_o \cdot e^{j\beta}$ are complex coefficients, b is the width of chip, Y_i and Y_o is the amplitude of tool oscillation and amplitude of surface waves respectively. Damping forces have the following comprehensive form in the proposed model:

$$\begin{aligned}
F_D &= |F_D| \cdot e^{j\gamma} \\
F_W &= |F_W| \cdot e^{j\delta}
\end{aligned}
\tag{5}$$

In equations (4) and (5) the indices α , β , γ , δ denote the phases of individual forces against the tool oscillations Y_i in direction „a“. Generally, these phases will be different for the direction „b“; thus later we must use other symbols φ , σ , ζ , ξ to denote them. It can be assumed that these phases are dependent on the tool geometry and on the cutting edge wear. It is obvious from the equations (4) that the vector of force F_i does not change its position

depending on the phase ε . The force of F_i has the size KbY_i and phase α in the complex plane. On the contrary, the force F_o has phase $(\beta+\varepsilon)$ against Y_i , so that when we change ε , the force F_o rotates in the complex plane. In the vector sum of the forces (F_i+F_o) , the F_o rotates around the endpoint of the vector F_i and its endpoint moves around the circle. In general, this circle lies in any quadrant of the complex plane. Its location depends on the phase α of the force F_i . See Fig. 5.

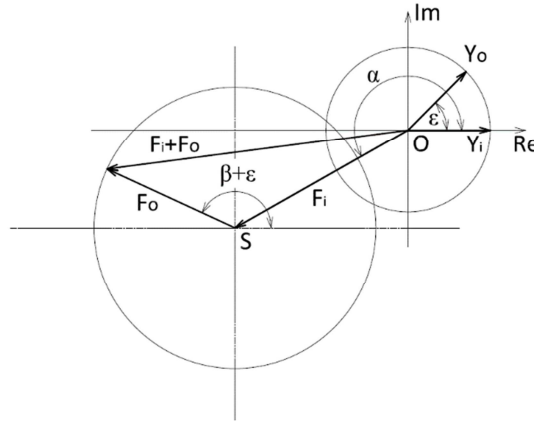


Fig. 5. View of the forces F_i , F_o and their summation (F_i+F_o) in the complex plane for a particular phase shift $\beta+\varepsilon$ of the force F_o

As proposed in the equation (2), the forces F_D and F_W influenced the overall forces F^a and F^b . As a result of the rotation of the phasor F_o , the overall forces F^a and F^b respond to a change of the phase ε by turning the force phasor in relation to the reference tool oscillations, i.e. in relation to the real axis of the complex plane. A gradual change in ε in the range of 0° to 360° thus causes the movement of the endpoint F^a and F^b , on a circle. This generates a sufficient number of measured points that can be fitted by a circle using the method of least squares and provides the centre and the radius of this circle.

To identify the components of the total forces, their properties will be used. We assume that the phase of the forces F_D and F_W are not dependent on the phase ε . Thus they will not affect the shape of the measured circle. Assuming for a moment that F_D and F_W are zero, the magnitude of F_i defines the distance of the centre S of the circle from the beginning of the complex plane and the magnitude of the force F_o defines the radius of the circle. See the red vectors F_i , F_o , (F_i+F_o) and circle in Fig. 6. We presume that the force F_D can be eliminated by reducing the steepness of the waves $Y_o(t)$, which can take place either at higher speeds or lower frequencies of tool excitation. The amplitude of the tool vibrations can be also used to influence the steepness of the waves. The force F_W can be eliminated by using a sharp tool. On the contrary, the force of F_D will be greatest at low speeds. The force F_W will be the greatest at higher, but defined, tool wear.

Substituting the equations (4) into the equation (2) and neglecting the force F_W we obtain (for the direction “a”):

$$F = K_o \cdot e^{j(\beta+\varepsilon)} \cdot b \cdot Y_i + (K_i \cdot e^{j\alpha} \cdot b \cdot Y_i + |F_D| \cdot e^{j\gamma})$$

which can be written:

$$F = F_o + (F_i + F_D) \quad (6)$$

These equations are depicted in Fig. 6 by the green vectors and circle. Obviously the vector sum $F_i + F_D$ defines the new position of the circle centre and the rotating vector F_o creates the green circle of the same radius as the previous red one. Thus the unknown force F_D can be identified through measuring both the red and green circles and fitting their centres. The vector difference of these centres defines F_D . Adding F_w , the analogical procedure can be applied. It is depicted by the blue vectors and circle in Fig. 6. This procedure applies to both directions “a” and “b”.

The method assumes that the differences between the phasors F^a or F^b , measured under different cutting conditions will identify the forces F_i , F_o , F_D and F_w with sufficient accuracy. A background of the proposed method can be found in [3].

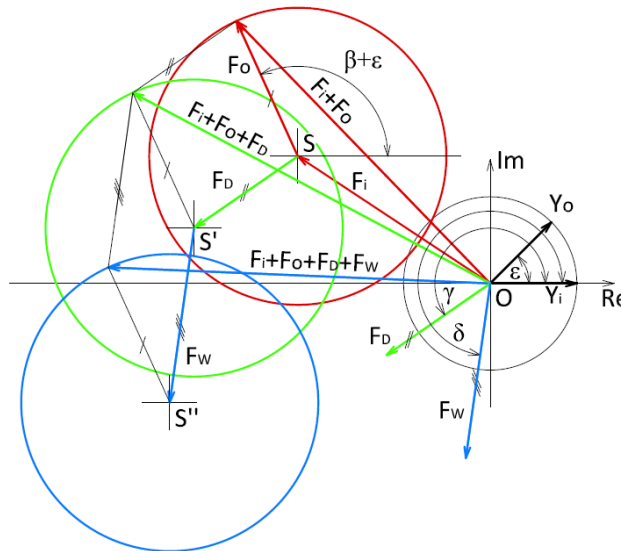


Fig. 6. Identification of the forces F_D and F_w for a particular phase shift $\beta + \epsilon$ of the force F_o .

To calculate the stability limit, the total forces F^a and F^b may be used. However, the result would be valid only for those cutting conditions under which these forces have been measured. In order to calculate the stability limit useful for a certain range of cutting conditions, it is necessary to identify all forces F^a_i , F^a_o , F^a_D and F^a_w , and also forces F^b_i , F^b_o , F^b_D and F^b_w and their dependencies on the variable cutting conditions.

4. PRINCIPLE OF EXPERIMENT

The experiment described below does not validate any model with reality. A validation in this sense cannot be done until we have identified all true forces acting in an unstable cut.

Based on this true force model, a stability diagram can be calculated and this can be validated with reality. The prepared experiment, described in the paper, will serve only to the identification of true forces.

To identify the dynamic cutting forces it is necessary to set the phase ε step by step. Naturally-induced self-excited vibration does not allow the setting up of the phase. Therefore, an artificial excitation of the cutting tool will be applied, so that the tool can create the waves $Y_o(t)$ on the workpiece. The phase ε will be set by change of wave length, which is given by the ratio of the frequency of workpiece rotation to the frequency of the tool excitation.

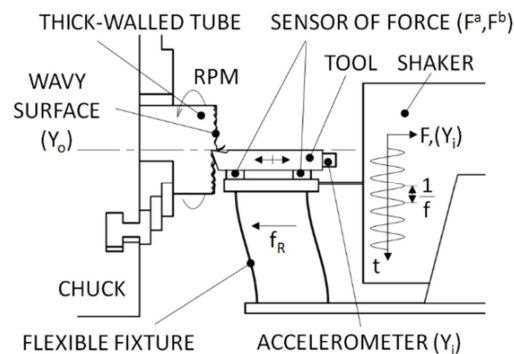


Fig. 7. Scheme of the measuring fixture

In order to create waves on the machined surface, tool excitation in a direction perpendicular to the machined surface will be used. The excitation will be repeated for various values of phase ε or frequencies of the actuator exciting signal so that the tip of vector F^a or F^b will describe a circle around a fixed centre S in complex plane.

The workpiece will have the shape of a thick-walled tube and will be face machined in the direction of the workpiece larger stiffness. The vibrating system will be formed by the tool. It must be ensured that such a system is able to vibrate only in the direction of the “ a ” axis, while it is very stiff in the direction of the “ b ” axis. The direction of tool vibration will be the same as the direction of the workpiece rotational axis. The principle scheme of the equipment can be seen in Fig. 7. It is essential to measure both the magnitude and the phase of F^a and F^b relative to excited tool vibration Y_i . Tool vibration Y_i needs to be scanned during machining.

A tool will be mounted in a flexible fixture. The tool will move lengthwise at a speed that will be determined by the feed per revolution f_R and it will machine the face of the tube clamped in a chuck. The tube will rotate at constant speed, in order to maintain constant cutting speed throughout the test. The thickness of the walls of the tube will determine the width of the chips, which will also remain unchanged throughout the test. Let us bear in mind that the feed per revolution defines the static components of cutting forces. The tool will be excited by means of a force shaker with a chosen frequency and amplitude (f , Y_i) during its feed. Thereby the created wavy surface (Y_o) will be cut off by the vibrating tool during the following revolution. This process will produce a variable depth of chip ($Y_o - Y_i$).

This depth will generate dynamic cutting force F^a and F^b , which will be recorded by proper force sensors. Waves on the workpiece surface will be shifted by a phase angle ε , against tool vibration Y_i . The phase will be determined by the ratio of the tool oscillation frequency and cutting speed. For each longitudinal crossing of the tool the ratio will be fixed. Throughout the test, the frequency of the tool oscillation will be gradually changed so that the phase shift ε between $Y_i(t)$ and $Y_o(t)$ will vary in the range of 0° to 360° .

For each chosen value of cutting speed and each frequency the measured amplitude and phase of the forces F^a and F^b will be drawn into the complex plane. The above-mentioned analysis of the results obtained (Fig. 3), will allow us to specify the dynamic coefficients of cutting forces. The same procedure will be repeated for the next cutting speed. Thus it will be possible to observe the behaviour of the force coefficients depending on cutting speed.

5. SUMMARY

The paper proposes a method of experimental identification of the complex forces that act during unstable orthogonal turning. The forces are considered as individual forces, not components of one force. The method assumes that at least four forces must be identified: the forces of inner and outer modulation, process damping force and damping force generated in the cut due to tool flank wear. All the forces are assumed to be complex and of different size and direction. This approach has not been investigated yet. The existing stability calculations consider only one dynamic cutting force decomposed into two orthogonal components. On the one hand, the assumption of the complex forces complicates the identification, but on the other hand it contributes very probably to an improvement of the stability limit prediction. The forces have to be identified experimentally, under the cutting conditions variable in proper ranges. The experimental validation of the proposed theory and method of the complex force identification will be a subject of a future investigation.

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