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METHODS OF COMPARISON OF SURFACE TEXTURE BASED ON FRACTAL DIMENSION AND HOTELLING'S T^2 TEST

Comparative analysis of the surface texture of machine parts can be successfully carried out using statistical tests. The paper presents a methodology of method used to compare the surface texture by applying Hotelling's T^2 test as well as a method used to evaluate surface topography by applying fractal dimension. The tests were carried out on samples produced with the use of face milling process for four types of materials. The following types of steel were used: 40HM, C45, NC6 and WCL. For each type of material, four areas were machined with the same machining parameters. Based on these results a decision was made whether the surfaces, despite the same machining conditions, were significantly different from each other. Furthermore, the analysis indicated that the fractal dimension enabled to characterise signal irregularities in quantitative and qualitative way.

1. INTRODUCTION

The dynamic development of science and technology implies continuous improvement requirements for quality of machine parts. This entails the need for much more precise machining and monitoring of the production process. Requirements set in the technological process, concerning the quality of a manufactured element and the pursuit of increasing durability of machines make diagnostics of the machining process a crucial part of each production process. It allows to detect undesirable phenomena that can lead to tool damage, changes in the quality of surface texture of manufactured parts, or their dimensional and shape accuracy. There are many methods for manufacturing of machine parts. Undoubtedly, machining is one of the most commonly used in the production process. Currently, about 70% of all components are produced using this method. One of the dominant methods of such production is milling.

Surface texture of machine parts consists of a series of periodic and non-periodic irregularities which constitute the geometrical surface structure, i.e. roughness, waviness and form. All these irregularities depend on many factors occurring during the machining process, among others machining parameters, technical condition of machine tool, positioning of cutting inserts in milling head, irregularities in cutting edge, material properties or human

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errors. Thus, it is not possible to obtain a homogeneous structure over the entire surface of an element. Surface texture is irregular, which may cause changes in the behaviour of mechanisms during cooperation of its parts. That is why there is a need to monitor and assess the changes in the structure of surface texture over the entire surface of machine elements [1–4]. When analysing the current state of the art, it must be noted that there are many methods used to evaluate the surface texture. The analysis may be based on statistical method, assessment of individual waviness or roughness parameters, as well as on a modern method of data processing, such as spline filter, morphological filters or wavelet analysis [5–10]. In this paper author proposes a new approach to characterisation of surface texture of machine parts. In this method, a two grids of points describing surfaces are compared using statistical tests. In the paper, Hotelling's test was adopted to assess surface texture. The application of T^2 test allows to compare two matrices and determine with a certain probability whether surfaces are similar to each other. Hotelling's test allows for a comparative analysis of surface textures of different machine parts, as well as a comparative analysis of measurement data obtained from different sections of an element. Such information may be useful from the point of view of diagnostic analysis of manufacturing processes.

2. STATISTICAL METHODS FOR COMPARISON OF SURFACE TEXTURES

2.1. THE CONCEPT OF COMPARISON OF SURFACES USING HOTELLING'S TEST

This test is a generalisation of Student's t-statistics (which can be applied to assess e.g. a 2D profile) for multidimensional space, including a 3D surface texture. The methodology was developed by Harold Hotelling, an American mathematical statistician and an influential economic theorist. He also developed and named the principal component analysis method widely used in finance, statistics and computer science. Hotelling's T^2 test compares the average value of vectors in two populations. The author used the test for comparison of four 3D surface irregularities created using the same machining parameters. In the analysis, the coefficients which described two selected surfaces were compared. Based on the results it was assessed that the surfaces were significantly different from each other [11, 12].

It is important to consider the test assumptions. There are three assumptions underlying Hotelling's T^2 test. The first is independence. The subjects from both populations are independently sampled. The next is normality. Both populations are multivariate normally distributed. The last is homogeneity of variance-covariance matrices. This assumption may be assessed using Bartlett's test. In this test the null hypothesis H_0: $\Sigma_1=\Sigma_2$ is tested against the alternative H_1: $\Sigma_1\neq\Sigma_2$. The Bartlett's statistic is expressed by L and the statistic value is calculated using the following formula (1):

$$L = c[(n_1 + n_2 - 2)\ln|S_p| - (n_1 - 1)\ln|\Sigma_1| - (n_2 - 1)\ln|\Sigma_2|] \quad (1)$$

where:

$$S_p = \frac{(n_1 - 1)\Sigma_1 - (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2} \quad (2)$$

$$c = 1 - \frac{2p^2 + 3p - 1}{6(p+1)}\left[\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} - \frac{1}{n_1 + n_2 - 2}\right] \tag{3}$$

The calculated value of the statistic L is compared with the critical value of χ^2 distribution on an assumed significance level α. The null hypothesis H_0 should be rejected if the resulting value is greater than the critical value.

$$L > \chi^2_{\frac{p(p+1)}{2},\alpha} \tag{4}$$

As a result of the measurement of the surface texture, a set of points distributed over the surface in the form of a regular grid (n_1 x p) was obtained. The following procedure was carried out.

For each surface profile a mean value was calculated.

$$\bar{x} = \frac{1}{n_1}\sum_{1}^{n_1} x_i \tag{5}$$

The next step was to create the variance-covariance matrix,

$$\Sigma_1 = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_i - \bar{x})(x_i - \bar{x})^T = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}\left(\begin{pmatrix} x_1 - \bar{x} \\ x_2 - \bar{x} \\ \dots \\ x_{n_1} - \bar{x} \end{pmatrix}\begin{pmatrix} x_1 - \bar{x} & x_2 - \bar{x} & \dots & x_{n_1} - \bar{x} \end{pmatrix}\right) =$$

$$\frac{1}{n_1 - 1}\sum_{i=1}^{n_1}\begin{pmatrix} (x_1 - \bar{x})(x_1 - \bar{x}) & (x_1 - \bar{x})(x_2 - \bar{x}) & \dots & (x_1 - \bar{x})(x_{n_1} - \bar{x}) \\ (x_2 - \bar{x})(x_1 - \bar{x}) & (x_2 - \bar{x})(x_2 - \bar{x}) & \dots & (x_2 - \bar{x})(x_{n_1} - \bar{x}) \\ \dots & \dots & \dots & \dots \\ (x_{n_1} - \bar{x})(x_1 - \bar{x}) & (x_{n_1} - \bar{x})(x_2 - \bar{x}) & \dots & (x_{n_1} - \bar{x})(x_{n_1} - \bar{x}) \end{pmatrix} \tag{6}$$

Similar calculation was performed for the second surface (n_2 x p).

$$\bar{x} = \frac{1}{n_1}\sum_{1}^{n_1} x_i \tag{7}$$

$$\Sigma_2 = \frac{1}{n_2 - 1}\sum_{i=1}^{n_2}(y_i - \bar{y})(y_i - \bar{y})^T \tag{8}$$

In case that the variance-covariance matrices are homogeneous, Hotelling's statistic is expressed by the following formula (9):

$$T^2 = (\bar{x} - \bar{y})^T V^{-1}(\bar{x} - \bar{y}) \tag{9}$$

where:

$$V = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2} \tag{10}$$

otherwise, the *V* value must be modified.

$$V = \frac{\Sigma_1}{n_1} + \frac{\Sigma_2}{n_2} \tag{11}$$

The final phase was to make the decision whether surfaces were significantly different from each other. Hotelling's statistic must be converted to Snedecor distribution using the following formula (12):

$$F = \frac{n_1 + n_2 - p - 1}{p(n_1 + n_2 - 2)} T^2 \sim F_{p, n_1 + n_2 - p - 1} \tag{12}$$

The obtained value was compared with the critical value. When the calculated value is lower than the critical value at the significance level, it should be recorded that the surfaces are not significantly different from each other.

$$F < F_{p, n_1 + n_2 - p - 1, \alpha} \tag{13}$$

2.2. FRACTAL DIMENSION

Comparative analysis of surface texture of machine parts can be carried out using fractal dimension. Fractal analysis is a mathematical method created in the 1960s by a Polish-born French and American mathematician Benoit B. Mandelbrot. Fractal dimension is defined as a measure of self-similarity. Fractals are used mainly to describe highly irregular signals. Surface texture is undoubtedly one of those. According to fractal geometry, it is possible to describe irregular lines, uneven planes and volumes that exist anywhere in nature. It enables to characterise signal irregularities in quantitative and qualitative way. Many researchers have applied fractal analysis to characterise surface texture. It was concluded that fractal analysis can be used to characterise the surface topography of samples produced with the use of various methods of manufacturing, as well as to predict mechanical and exploitative properties [13]. In particular, the application of fractal dimension to a surface produced by machining process can reflect the characteristic properties of the surface, which are impossible to detect using other surface roughness parameters. For real surfaces, the fractal dimension value is in the range of 2 to 3, while for an uncomplicated, smooth surface structure the parameter value is close to 2, and for a surface characterised by a chaotic distribution of irregularities the value is close to 3. In this paper the author used fractal dimension which is defined by the following formula (14) [14, 15]:

$$Sfd = \lim_{\varepsilon \to 0} \frac{\log N(A, \varepsilon)}{\log \frac{1}{\varepsilon}} \tag{14}$$

where: $N(A,\varepsilon)$ defines the minimum number with a radius $\varepsilon > 0$, which is required to cover set *A*.

The tested parameter, which defines fractal dimension was also analysed in terms of the value calculated for various points of measurement. For this purpose, the coefficient of variation, which is a ratio of the standard deviation to the mean value, was determined using the following formula (15)

$$V = \frac{s}{\bar{x}} 100\% \quad (15)$$

where: s – standard deviation, $\bar{x}$ – mean value.

3. RESULTS OF CALCULATIONS

The research was carried out on samples produced with the use of the face milling process for four types of materials. Four types of steel were selected: 40HM, C45, NC6 and WCL. AVIA VMC800 machining centre was employed to carry out the cutting process. Each type of steel was machined with specified parameters (cutting speed – v_c, axial depth of cut – a_p, feed per tooth – f_z):

– 40HM steel (42CrMo4): v_c = 300 m/min, a_p = 0.2 mm, f_z = 0.16 mm/tooth;
– C45 steel: v_c = 260 m/min, a_p = 0.2 mm, f_z = 0.1 mm/tooth;
– NC6 steel: v_c = 300 m/min, a_p = 0.2 mm, f_z = 0.02 mm/tooth;
– WCL steel (X37CrMoV51): v_c = 300 m/min, a_p = 0.2 mm, f_z = 0.04 mm/tooth.

Measurements of surface texture were carried out using non-contact profilometer Talysurf CCI. In Figs. 1, 2 an isometric view of the surfaces is shown.

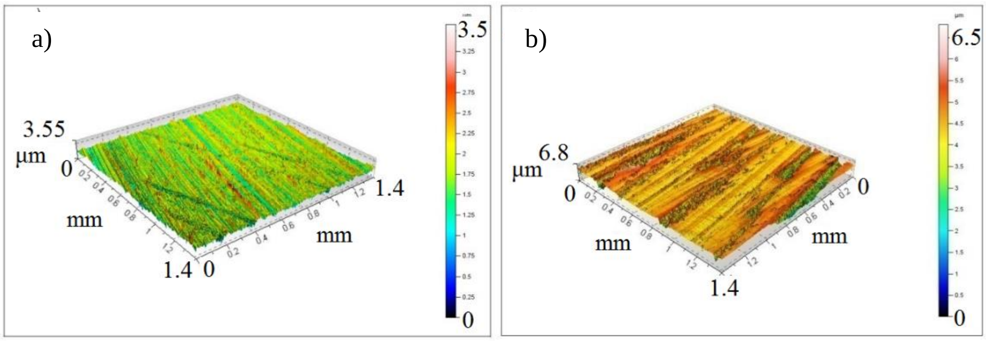

Fig. 1. Isometric view of the surface a) 40HM steel b) C45 steel

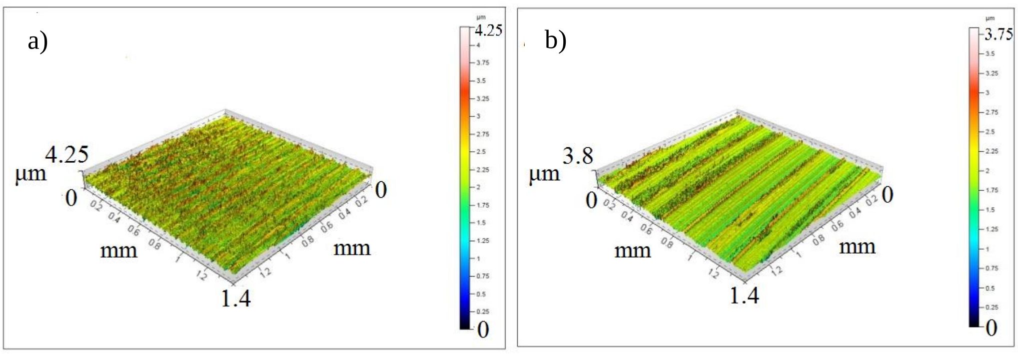

Fig. 2. Isometric view of the surface a) NC6 steel b) WCL steel

In the first research phase, the possibility of application of Hotelling‘s T^2 test to compare two surfaces was assessed. The calculations were performed in authorial computer procedures coded in MATLAB software. In order to carry out Hotelling’s test, the surface was measured in four independent areas. The obtained surfaces were compared in pairs. The results of calculations are summarised in Tables 1–4. The calculated values were compared with the critical value, with the significance level $\alpha = 0.05$, $F_{p,n1+n2-p-1,\alpha} = 1.171$ according to the equation (13).

On this basis, the decision was made to support or reject the hypothesis that the two surfaces are not significantly different from each other. The symbol “+” indicates that a pair of surfaces fulfilled the test condition (13). Otherwise, the symbol “–” is put in the tables.

Table 1. Results of calculations for 40HM steel

1×2	1×3	1×4	2×3	2×4	3×4
0.108	0.205	0.198	0.163	0.272	0.415
+	+	+	+	+	+

Table 2. Results of calculations for C45 steel

1×2	1×3	1×4	2×3	2×4	3×4
1.926	1.211	0.718	1.118	2.904	3.025
–	–	+	+	–	–

Table 3. Results of calculations for NC6 steel

1×2	1×3	1×4	2×3	2×4	3×4
0.196	0.452	0.577	0.596	0.703	0.532
+	+	+	+	+	+

Table 4. Results of calculations for WCL steel

1×2	1×3	1×4	2×3	2×4	3×4
3.233	2.778	2.190	1.287	2.112	1.697
–	–	–	–	–	–

Based on Hotelling’s T^2 test, at the assumed significance level, it should be noted that the machining parameters have a significant impact on the surface texture. Table 1 shows the test results for 40HM steel. Based on the statistical tests, it was stated that the geometric structure of the surface at the measured areas showed no significant difference between particular areas of surface. Similar results were obtained for NC6 steel. The critical value did not exceed the critical value for each pair of surfaces. According to Table 3, it must be recorded that the surface was not significantly different across the element. The results of the calculations for the other two types of steel are presented in Table 2 and Table 4. The analysis carried out for C45 steel, using Hotelling’s test, showed that there were detected the surfaces with are in pairs similar to each other. According to the data in Table 4, surface No. 1 and surface No. 4, at the assumed significance level, were not significantly different from each other, similarly to surfaces No. 2 and No. 3. However, in other cases, the obtained values of T^2 statistics were higher than the assumed critical value. Analysing the tables it should be noted that the surface texture of WCL steel was characterised by low homogeneity. All of the pairs of surfaces fulfilled the condition (13). The obtained values exceeded the critical value at the assumed significance level. For this kind of material and machining parameters, the test should be repeated. A greater number of surface areas and their placement on the surface should be considered.

In order to characterise surface irregularities in quantitative and qualitative way analyses were carried out using fractal dimension. The calculated values of fractal dimension are shown in Table 5. For almost all surfaces, the obtained values were greater than 2.5. This indicates that the tested surfaces were characterised by a chaotic distribution of irregularities of the 3D surface. The calculated values of the coefficient of variation showed that for 40HM steel and NC6 steel the surfaces were almost homogeneous in the tested areas of the sample. Different results were obtained for the other two materials. For C45 steel, the obtained fractal dimension values for surfaces No. 1 and No. 4 were equal. The result was confirmed by the obtained values of Hotelling's test. According to the T^2 test, it was shown that the character of the surface texture is similar in this two areas. High variability of values was obtained for WCL steel. According to Table 5, it can be stated that particular areas are not statistically similar to each other.

Table 5. Research results of fractal dimension

	40HM	C45	NC6	WCL
1	2.57	2.42	2.61	2.70
2	2.58	2.57	2.60	2.59
3	2.59	2.54	2.61	2.77
4	2.59	2.42	2.61	2.65
V, %	0.37	3.17	0.19	2.85

When analysing the data contained in all the above tables, it should be noted that similar results were obtained for both methods. This fact confirms that Hotelling's T^2 test can be used to analyse surface textures.

4. CONCLUSION

Comparative analysis of surface texture of machine parts can be successfully carried out using statistical tests. The concept of using Hotelling's T^2 statistical test is a new method for comparison of two surfaces. The paper presents the principles of the use of Hotelling's test. The paper also presents the results of the possibility of adapting these statistics in the metrology of surface texture, for four types of materials. For each material four independent areas were determined, which were then analysed. Geometric structure of the surfaces were analysed in pairs. After analysing the results it can be concluded that the best results were achieved for 40HM steel and NC6 steel. There was no area which was significantly different than other areas on the surface. The surface texture was homogeneous over the entire surface. Research results obtained for the C45 steel suggested that the surface of the sample was characterised by similar areas on the surface, however, it was not homogeneous. Surfaces which were similar in pairs were noted. The material, for which the results did not fulfil the test conditions was WCL steel. The test results for each pair significantly exceeded the critical value for the assumed significance level. It must be inferred that the surface is significantly different across the sample. This situation can be caused by improperly selected machining parameters.

The evaluation of surface was also carried out using fractal dimension. For real surfaces, the fractal dimension value is in the range of 2 to 3. For almost all surfaces, the obtained values were greater than 2.5. This indicates that the tested surfaces were characterised by a chaotic distribution of irregularities of the 3D surface. The obtained values of the coefficient of variation show that for two materials (WCL and C45), the variability of geometrical structure of the surface was noticeably significant. For none of the tested surfaces for WCL steel, statistical analysis showed no similarity between the surfaces.

Analysing the research results it should be noted that Hotelling's T^2 test and fractal dimension are suitable methods to evaluate surface texture. Both methods are sensitive to surface texture changes. When analysing the research results it should be noted that similar results were obtained for both methods.

Future research may be extended by experimental verifications whether statistical similarity is related with functional similarity.

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